EXPERIMENT 10

AIM

To find the focal length of a convex lens by plotting graphs between u and v or between $1/u$ and $1/v$.

APPARATUS AND MATERIAL REQUIRED

An optical bench, two sharp-edged needle (pins), convex lens of less than 20 cm focal length, three uprights (with clamps), index needle (may be a knitting needle), metre scale and spirit level.

TERMS AND DEFINITIONS

1. *Principal axis* of a lens is the line joining centre of curvature of the two surfaces.

2. *Optical centre* is the point, through which a ray passes undeviated through the lens.

3. *Principal focus* is the point where rays parallel to the principal axis focus after passing through the lens (convex) or appear to come from after passing through the lens (concave).

4. *Focal length* is the distance between optical centre of lens and focus.

5. *Intercepts of a graph*: If a graph cuts x-axis and y-axis, then lengths between origin and points of interception are intercepts of the graph.

GRAPHICAL METHOD FOR FINDING THE POSITION OF AN IMAGE FORMED BY THIN LENSES

The image formed by a thin lens can be located using graphical method by considering refraction of rays emanating from each point on the object. However, it is convenient to choose any two of the following three rays (Fig. E 10.1).

1. A ray from the tip of the object parallel to the principal axis of the lens that passes through the second principal focus F′ (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F after refraction.

2. A ray from the tip of the object incident at the optical centre, passes through the lens undeviated. This is because near its centre, the lens behaves like a thin glass slab.

3. A ray of light from the same point on the object that passes through the first principal focus F (for a convex lens) or appearing to pass through F′ (for a concave lens) emerges parallel to the principal axis after refraction.

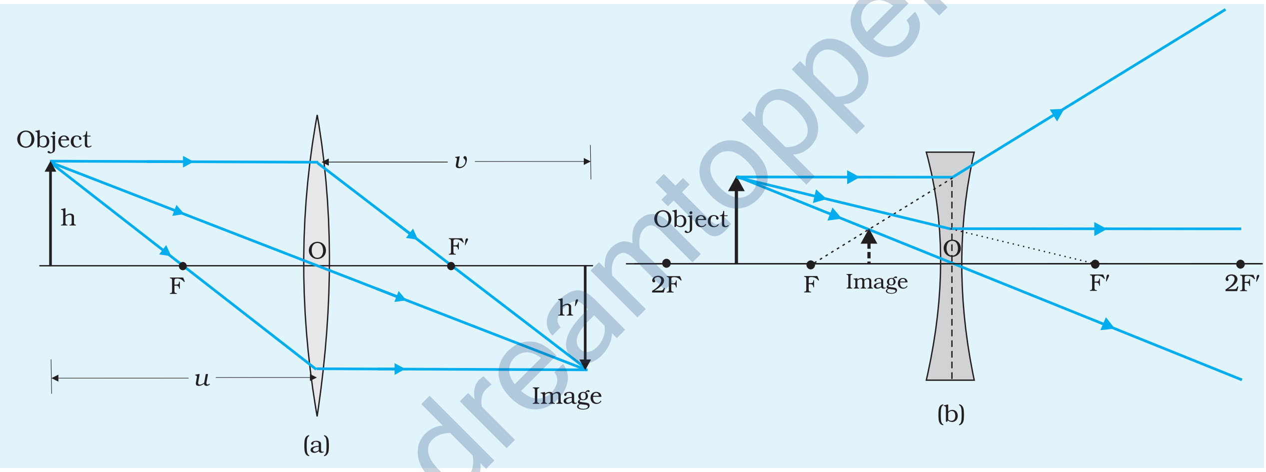

Fig. E 10.1 *Ray tracing for locating the image formed by (a) convex lens and (b) concave lens.*

PRINCIPLE

For an object placed at a distance u from the optical centre of a thin convex lens of focal length f, a real and inverted image is formed on the other side of the lens at a distance v from the optical centre. The relation between these distances is:

(E 10.1)
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

According to the new cartesian sign convention (*see Physics Textbook, NCERT, 2007, Class XII, Part-II p. 311,*) u is negative but v is positive [Fig. E 10.2 (a) and (b). Therefore the Eq. (E 10.1) takes the following form for magnitudes of u and v.

(E 10.2)
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\text{or,}\quad f = \frac{uv}{u+v} \qquad \text{(E 10.3)}$$

In this result the positive values of u and v are substituted.

Eq. (E 10.2) shows that $\frac{1}{v}$ versus $\frac{1}{u}$ graph is a straight line of negative slope. If $\frac{1}{v}$ equals zero or $\frac{1}{u}$ equals zero, then respectively $\frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{f}$. The intercepts of the graph on both axes are $\frac{1}{f}$. Graph of u versus v is a hyperbola. When $u = v$, then each equals $2f$. Eq. (E 10.3) shows that values of u and v are interchangeable.

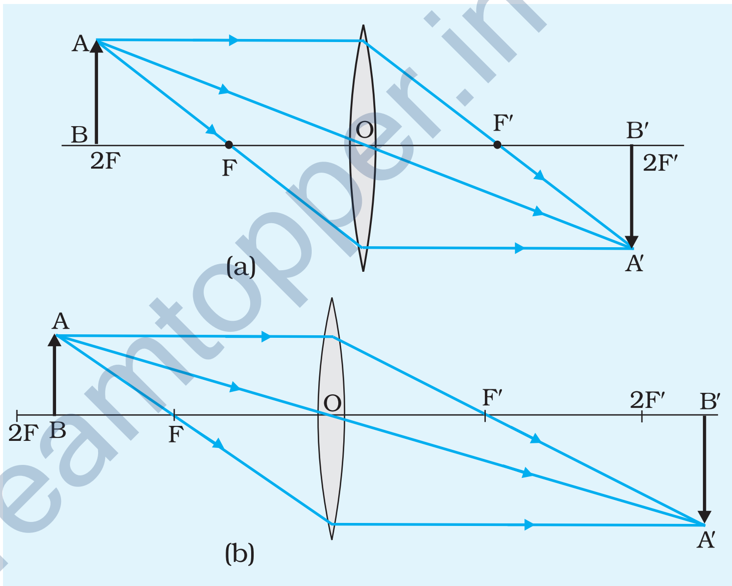

Fig. E 10.2 (a), (b) *Formation of image by a convex lens (a)* $u = 2f$ *and (b)* $2f > u > f$.

When an object (say, a pin) is placed in front of a thin convex lens at a distance equal to $2f$, a real and inverted image of same size as that of the object is formed at a distance equal to $2f$ on the other side of the lens [Fig. E 10.2(a)]. If the object's position lies in between distance $2f$ and distance f from the optical centre of the lens then a real, inverted and magnified image is formed at a point beyond $2f$ from the optical centre on the other side of the lens [Fig. E 10.2(b)].

Thus, by measuring the distances u and v, the focal length of the convex lens can be determined using Eq. (E 10.3). The focal length of the lens may also be determined by plotting graphs between u and v or between $1/u$ and $1/v$.

PROCEDURE

1. Obtain approximate value of the focal length of the thin convex lens by focusing the image of a distant object. It can be found by obtaining a sharp image of the Sun or a distant tree on a screen, say a plane wall, or a sheet of paper placed on the other side of the lens and measuring the distance between the lens and the image with a scale. This distance is a rough estimate of the focal length, f of the convex lens.

Note: Do not look at the image of Sun directly as it may hurt your eyes.

2. Place the optical bench on a rigid table or on a platform, and using the spirit level, make it horizontal with the help of levelling screws provided at the base of the bench.

3. Clamp the convex lens on an upright and mount it vertically almost near to the middle of the optical bench such that its principal axis is parallel to the optical bench. In this position, the lens would lie in a plane perpendicular to the optical bench.

4. For the determination of the index correction, bring a mounted pin close to the lens. Adjust the index needle (a sharp-edged knitting needle would also serve the purpose) horizontally such that its one end touches one of the curved surfaces of the lens and the other end touches the tip of the pin. Note the positions of the two uprights on the scale provided on the optical bench. The difference of the two would give the *observed length* of the index needle. The *actual length* between the tip of the pin and optical centre O would be length of the index needle (as measured by a scale) plus half of the thickness of the lens because optical centre of a double convex lens with surfaces of equal curvature is at its geometrical centre. The difference of the two lengths is the *index correction*. Find index correction for both the pins.

5. Place the vertically mounted sharp pins P and P′ (Fig. E 10.3) on left and right hand sides of the lens respectively. Adjust the pins P and P′ so that the heights of the tips of these pins become equal to the height of the optical centre O of the lens from the base of the optical bench. Let the pin P (placed on left hand side of the lens) be the object pin and the pin P′ (lying on right hand side) be the image pin. Put a small piece of paper on one of the pins (say on image pin P′) to differentiate it from the object pin P′.

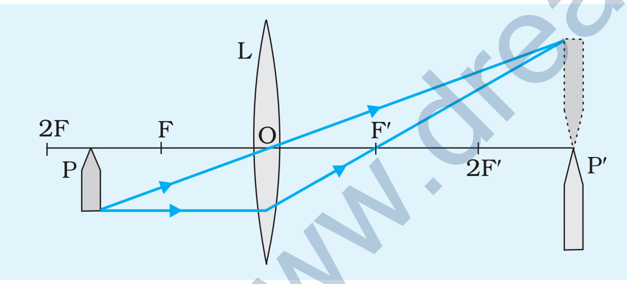

Fig. E 10.3 *The ray diagram for finding the focal length of a convex lens.*

6. Displace the object pin P (on left side of the lens) to a distance slightly less than $2f$ from the optical centre O of the lens (Fig. E 10.3). Locate the position of the real and inverted image on the other side of the lens above the image pin P′.

7. Using the method of parallax, adjust the position of the image pin P′ such that the image of the object pin P coincides with the image pin P′.

Note: As the value of u changes from $2f$ to f, v changes from $2f$ to infinity. Since the values of u and v are interchangeable, i.e., the object and image are two conjugate points, therefore it is clear that complete range of values for both u and v between f and infinity are obtained for a movement of the object pin over the range $2f$ to f.

8. Note the upright position of the object pin, convex lens and image pin on the optical bench and record the readings in an observation table.

9. Move the object pin P closer to the optical centre O of the lens (say by 2 cm or 3 cm). Repeat the experiment and record at least six sets of readings for various distances of object pin between f and $2f$ from the lens.

OBSERVATIONS

1. Approximate focal length of the convex lens = ... cm

2. Length of the index needle as measured by the metre scale, L_0 =... cm

3. Thickness of the thin convex lens (given), t = ... cm

4. Actual length between the optical centre O of the lens and tip of the pin, $l_0 = L_0 + t/2$ = ... cm

5. Observed length of the index needle, l'_0 = Distance between the centre of convex lens and tip of the object pin

   = Position of lens upright – position of object pin upright on the scale.

   = ... cm – ... cm =...cm

6. Index correction for object distance, $e_o = l_0 - l'_0$ = cm; similarly for image pin, $e_i = l_i - l'_i$ = ...cm

Table E 10.1: Determination of *u*, *v* and *f*

Sl. No.	Lens upright a (cm)	Object pin upright b (cm)	Image pin upright c (cm)	Observed $u = a - b$ (cm)	Observed $v = a - c$ (cm)	Corrected u = Observed $u + e_o$ (cm)	Corrected v = Observed $v + e_i$ (cm)	$\frac{1}{u}$ cm^{-1}	$\frac{1}{v}$ cm^{-1}	$f = \frac{uv}{u+v}$ cm	Δf (cm)
1											
2											
--											
6											
									Mean		

CALCULATIONS

A. Calculate the corrected values of u and v. Compute the value of f. using Eq. (E 10.3). Tabulate them in the table and find the mean value of the focal length of the given convex lens.

Error

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

or $$\frac{\Delta f}{f^2}=\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}$$

$$\Delta f = f^2\left[\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right]$$

Maximum of six values of Δf is to be reported with the result as the experimental error.

CALCULATION OF FOCAL LENGTH BY PLOTTING GRAPHS

(A detailed method of plotting graphs has been illustrated in Chapter 1 Article 1.7 (p. no. 15).

B. $u - v$ *Graph:* Take u along X-axis and v along Y-axis. Scales of x- and y-axis should be same. Draw a hyperbola curve for various values of u and v (Fig. E 10.4). Note that six sets of readings for u between f and $2f$, give you 12 points on the graph by interchanging values of u and v.

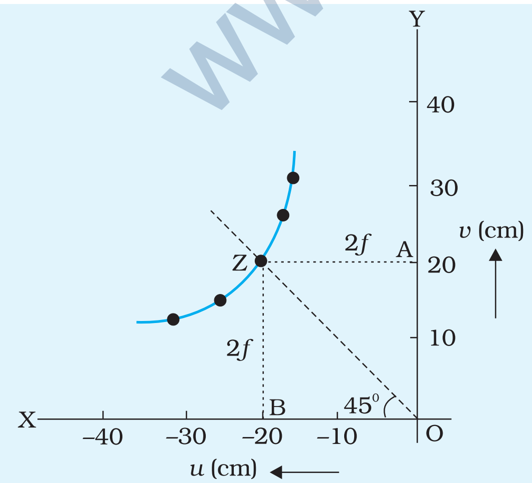

Fig. E 10.4 u versus v graph for convex lens

The point $u = 2f$; $v = 2f$ is shown as point Z on $u - v$ graph (Fig. E 10.4). The point Z is the point of intersection of a line OZ bisecting the angle ∠XOY with hyperbola. Draw two lines AZ and BZ perpendicular to Y- and X-axis, respectively. The lengths AZ and BZ are both equal to distance $2f$. Thus by plotting the $u - v$ graph, the focal length of the lens can be obtained.

Distance OA (= $2f$) on Y-axis = ... cm

Distance OB (= $2f$) on X-axis = ... cm

Mean focal length of the convex lens, f

= = ... cm

C. $1/u - 1/v$ graph: Draw a straight line graph by plotting $1/u$ along the X-axis and $1/v$ along the Y-axis (Fig. E 10.5). Both the intercepts OA′ (on Y-axis) and OB′ (on X-axis) will be equal to distance $1/f$.

Intercept OA′ (= $1/f$) on Y-axis =...cm^{-1}

Intercept OB′ (= $1/f$) on X-axis =...cm^{-1}

Mean focal length (f) of the convex

$$\text{lens} = \frac{2}{OA' + OB'} = \text{...cm.}$$

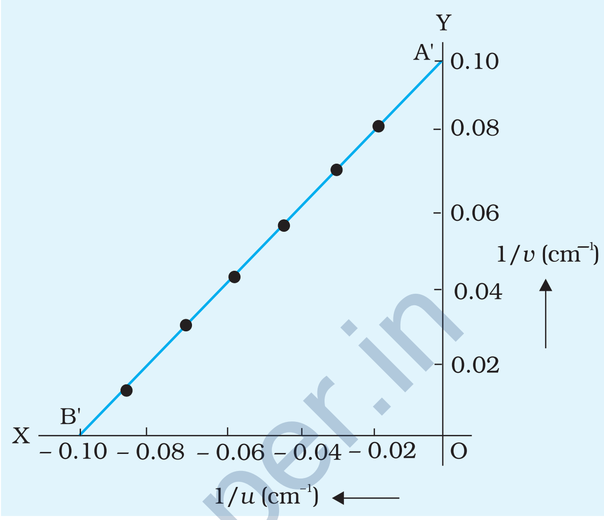

Fig. E 10.5 *$1/u$ versus $1/v$ graph for a convex lens (not to scale).*

RESULT

The focal length of the given converging thin convex lens:

(i) from calculations as shown in Observation Table E10.1 $f \pm \Delta f$ = ...cm (here f is mean value of the focal length)

(ii) from $u - v$ graph =...cm, and

(iii) from $1/u - 1/v$ graph =...cm.

PRECAUTIONS

1. The uprights supporting the optical elements should be rigid and mounted vertically.
2. The aperture of the lens should be small otherwise the image formed will not be distinct.
3. Eye should be placed at a distance more than 25 cm from the image needle.
4. An error may arise in the observations if the top of the optical bench is not horizontal and similarly if the tips of pins and optical centre of the lens are not at the same horizontal level.
5. The image and object needles should not be interchanged during the performance of the experiment, as this may cause change in index corrections for object distance and image distance.
6. The tip of the inverted image of the object needle must touch the

tip of the image needle and must not overlap. This should be ensured while removing the parallax.

7. The general instructions to be followed in all optical bench experiments (as given in the description of optical bench) must be taken care of.

8. The corrected values of the distances u and v must be put in the formula for calculating f and then a mean of f should be taken. Calculations for f must not be made using the mean values of u and v.

SOURCES OF ERROR

1. The uprights may not be vertical.

2. Parallax removal may not be perfect.

3. If the knitting needle or index rod for finding index correction is not sharp like a needle, its length may not be accurately found on scale.

DISCUSSION

In plotting $1/v$ versus $1/u$ graph, if scales for the two axes are not same, then the straight line graph may (rather will) not be at 45° to x-axis. This may result in confusions and error in drawing the graph. Keeping the scale same and drawing the best fit graph at 45° to x-axis is the best method. Then, due to inherent errors in measurement $1/f$ on both axes may be a bit too large or a bit too small.

SELF ASSESSMENT

1. Draw the ray diagram for image formation in case of a convex lens for position of object varying from infinity to optical centre.

2. What are the differences between the image formed by a convex lens and a concave lens?

3. How does the focal length of a thick convex lens differ from that of a thin lens?

4. How can you recognise a convex lens, a circular glass slab and a concave lens, without touching them?

5. Where does the centre of curvature of the plane surface of a plano-convex lens lie?

6. Define the principal axis of a plano-convex lens?

7. How does the focal length of a convex lens change if it is dipped in water?
8. What is the relation between focal length and radius of curvature of a plano-convex lens?
9. Can a virtual image produced by a lens be inverted?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Draw a graph by plotting uv along y-axis and $u + v$ along x-axis. Determine focal length f of the convex lens from the slope.
2. **EFFECT OF MEDIUM ON FOCAL LENGTH**

   You have an aquarium, an open window at some distance from it, and a magnifying glass of 50 mm diameter. With your 30 cm scale find the rough focal length of the magnifying glass in air. Then dip it in water by left hand and a white plastic bag (folded with a 5 cm × 5 cm card in it to make a white screen) by right hand. Focus image of a distant object on the screen by adjusting the position of the screen. Is the rough focal length in water bigger or smaller than that in air? Let a friend measure the focal length in water and find the ratio of the two.
3. **MEASURING LENGTH OF THE FILAMENT OF A CLEAR ELECTRIC LAMP**

   You cannot put the scale behind and in contact with the filament to measure its length. Of course you can do this measurement by vernier microscope. But can it be done by a simple convex lens and a scale only? You can also add things like clamp stand etc, as per needs of the experiment. Can you also measure the dark (cool) gap between adjacent segments of the filament of the lamp?