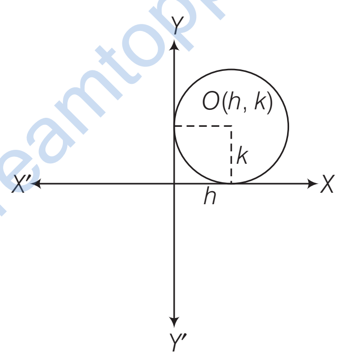
11

Conic Sections

Short Answer Type Questions

Q. 1 Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

Sol. Given that radius of the circle is a *i.e.*, $(h, k) = (a, a)$



So, the equation of required circle is

$$(x - a)^2 + (y - a)^2 = a^2$$

$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = a^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay + a^2 = 0$$

Q. 2 Show that the point (x, y) given by $x = \dfrac{2at}{1 + t^2}$ and $y = \dfrac{a(1 - t^2)}{1 + t^2}$ lies on a circle.

Sol. Given points are $x = \dfrac{2at}{1 + t^2}$ and $y = \dfrac{a(1 - t^2)}{1 + t^2}$.

$$\because \quad x^2 + y^2 = \frac{4a^2t^2}{(1 + t^2)^2} + \frac{a^2(1 - t^2)^2}{(1 + t^2)^2}$$

$$\Rightarrow \quad \frac{1}{a^2}(x^2 + y^2) = \frac{4t^2 + 1 + t^4 - 2t^2}{(1 + t^2)^2}$$

$$\Rightarrow \quad \frac{1}{a^2}(x^2 + y^2) = \frac{t^4 + 2t^2 + 1}{(1+t^2)^2}$$

$$\Rightarrow \quad \frac{1}{a^2}(x^2 + y^2) = \frac{(1+t^2)^2}{(1+t^2)^2}$$

$$\Rightarrow \quad x^2 + y^2 = a^2, \text{ which is a required circle.}$$

Q. 3 If a circle passes through the points (0, 0), (a, 0) and (0, b), then find the coordinates of its centre.

Thinking Process

General equation of the circle passing through the origin is $x^2 + y^2 + 2gx + 2fy = 0$. *Now, satisfied the given points to get the values of g and f. The centre of the circle is* $(-g, -f)$.

Sol. Let the equation of circle is

$$x^2 + y^2 + 2gx + 2fy = 0 \quad \text{... (i)}$$

Since, this circle passes through the points A (0, 0), B (a, 0) and C (0, b).

$$\therefore \quad a^2 + 2ag = 0 \quad \text{... (ii)}$$

$$\text{and} \quad b^2 + 2bf = 0 \quad \text{... (iii)}$$

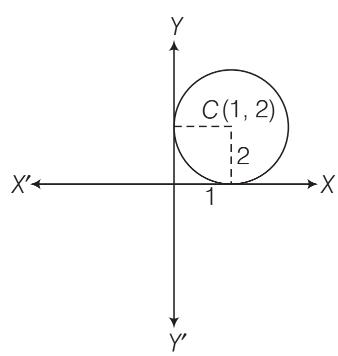
From Eq. (ii), $a + 2g = 0 \Rightarrow g = -a/2$

From Eq. (iii), $b + 2f = 0 \Rightarrow f = -b/2$

Hence, the coordinates of the circle are $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Q. 4 Find the equation of the circle which touches X-axis and whose centre is (1, 2).

Sol. Given that, centre of the circle is (1, 2).



$$\because \quad \text{Radius} = 2$$

So, the equation of circle is

$$(x - 1)^2 + (y - 2)^2 = 2^2$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 4y + 4 = 4$$

$$\Rightarrow \quad x^2 - 2x + y^2 - 4y + 1 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$$

Q. 5 If the lines $3x + 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

Thinking Process

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by,

i.e., $d = \left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$. *Use this formula to solve the above problem.*

Sol. Given lines,

$$3x - 4y + 4 = 0 \quad \text{.. (i)}$$

$$6x - 8y - 7 = 0$$

or $$3x - 4y - 7/2 = 0 \quad \text{...(ii)}$$

It is clear that lines (i) and (ii) parallel.

Now, distance between them *i.e.*,

$$d = \left|\frac{4 + 7/2}{\sqrt{9 + 16}}\right| = \left|\frac{\frac{8 + 7}{2}}{5}\right| = 3/2$$

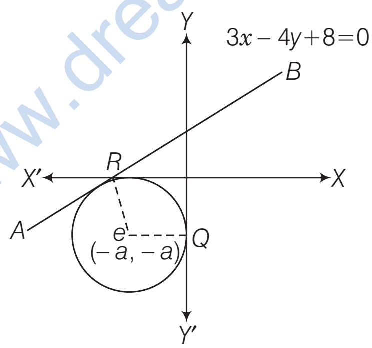
$\therefore$ Distance between these line = Diameter of these circle

$\therefore$ Diameter of the circle = 3/2

and radius of the circle = 3/4

Q. 6 Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

Sol.



Let a be the radius of the circle. Then, the coordinates of the circle are $(-a, -a)$. Now, perpendicular distance from C to the line AB = Radius of the circle

$$d = \left|\frac{-3a + 4a + 8}{\sqrt{9 + 16}}\right| = \left|\frac{a + 8}{5}\right|$$

$\because$ $$a = \pm\left(\frac{a + 8}{5}\right)$$

Taking positive sign, $$a = \frac{a + 8}{5}$$

$\Rightarrow$ $$5a = a + 8$$

$\Rightarrow$ $$4a = 8 \Rightarrow a = 2$$

Taking negative sign, $$a = \frac{-a - 8}{5}$$

$\Rightarrow$ $$5a = -a - 8$$

$\Rightarrow$ $$6a = -8 \Rightarrow a = -4/3$$

But $a \neq -4/3$

$\because$ $a = 2$

So, the equation of circle is

$$(x+2)^2 + (y+2)^2 = 2^2 \quad [\because a = 2]$$

$\Rightarrow$ $x^2 + 4x + 4 + y^2 + 4y + 4 = 4$

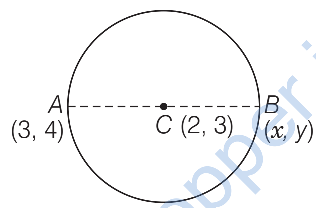
$\Rightarrow$ $x^2 + y^2 + 4x + 4y + 4 = 0$

Q. 7 If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of the diameter.

Thinking Process

First of all get the centre of the circle from the given equation, then find the mid-point of the diameter of the circle.

Sol. Given equation of the circle is



$$x^2 + y^2 - 4x - 6y + 11 = 0.$$

$\therefore$ $2g = -4$ and $2f = -6$

So, the centre of the circle is $(-g, -f)$ *i.e.*, (2, 3).

Since, the mid-point of AB is (2, 3).

Then, $2 = \dfrac{3 + x_1}{2}$

$\Rightarrow$ $4 = 3 + x_1$

$\therefore$ $x_1 = 1$

and $3 = \dfrac{4 + y_1}{2}$

$\Rightarrow$ $6 = 4 + y_1 \Rightarrow y_1 = 2$

So, the coordinates of other end of the diameter will be (1, 2).

Q. 8 Find the equation of the circle having (1, – 2) as its centre and passing through $3x + y = 14$, $2x + 5y = 18$.

Sol. Given that, centre of the circle is (1, – 2) and the circle passing through the lines

$$3x + y = 14 \quad \text{... (i)}$$

and $2x + 5y = 18$ ...(ii)

From Eq. (i) $y = 14 - 3x$ put in Eq. (ii), we get

$$2x + 70 - 15x = 18$$

$\Rightarrow$ $-13x = -52 \Rightarrow x = 4$

Now, $x = 4$ put in Eq. (i), we get

$$12 + y = 14 \Rightarrow y = 2$$

Since, point (4, 2) lie on these lines also lies on the circle.

$\therefore$ Radius of the circle $= \sqrt{(4-1)^2 + (2+2)^2}$

$= \sqrt{9 + 16} = 5$

Now, equation of the circle is

$$(x-1)^2 + (y+2)^2 = 5^2$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 + 4y + 4 = 25$$

$$\Rightarrow \quad x^2 + y^2 - 2x + 4y - 20 = 0$$

Q. 9 If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.

Sol. Given equation of circle,

$$x^2 + y^2 = 16$$

$\therefore$ Radius = 4 and centre = (0, 0)

Now, perpendicular from (0, 0) to line $y = \sqrt{3}\,x + k =$ Radius of the circle

$$\left|\frac{0 - 0 + k}{\sqrt{3+1}}\right| = 4$$

Since the distance from the point (m, n) to the line $Ax + By + k = 0$ is $d = \left|\frac{Am + Bn + C}{A^2 + B^2}\right|$

$$\Rightarrow \quad \pm\frac{k}{2} = 4$$

$$\therefore \quad k = \pm 8$$

Q. 10 Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and has double of its area.

Sol. Given equation of the circle is

$$x^2 + y^2 - 6x + 12y + 15 = 0 \qquad \text{...(i)}$$

$$\therefore \quad 2g = -6 \Rightarrow g = -3$$

$$2f = 12 \Rightarrow f = 6$$

and $\quad c = 15$

$$\therefore \quad \text{Centre} = (-g, -f) = (3, -6)$$

So, the centre of the required circle will be (3, – 6). [since, the circles are concentric]

Radius of the given circle

$$= \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{9 + 36 - 15} = \sqrt{30}$$

Let radius of the required circle $= r_1$

$$\therefore \quad 2 \times \text{Area of the given circle} = \text{Area of the required circle}$$

$$\Rightarrow \quad 2\,[\pi(\sqrt{30})^2] = \pi r_1^2$$

$$\Rightarrow \quad 60 = r_1^2$$

$$\Rightarrow \quad r_1 = \sqrt{60}$$

$$\therefore \quad \sqrt{g^2 + f^2 - c} = \sqrt{60}$$

$$\Rightarrow \quad 9 + 36 - c = 60$$

$$\Rightarrow \quad c = -15$$

So, the required equation of circle is $x^2 + y^2 - 6x + 12y - 15 = 0$.

Q. 11 **If the latusrectum of an ellipse is equal to half of minor axis, then find its eccentricity.**

Sol. Consider the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$\therefore$ Length of major axis $= 2a$

Length of minor axis $= 2b$

and length of latusrectum $= \frac{2b^2}{a}$

Given that, $\frac{2b^2}{a} = \frac{2b}{2}$

$\Rightarrow \quad a = 2b \Rightarrow b = a/2$

We know that, $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad \left(\frac{a}{2}\right)^2 = a^2(1 - e^2)$

$\Rightarrow \quad \frac{a^2}{4} = a^2(1 - e^2)$

$\Rightarrow \quad 1 - e^2 = \frac{1}{4}$

$\Rightarrow \quad e^2 = 1 - \frac{1}{4}$

$\therefore \quad e = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Q. 12 **If the ellipse with equation $9x^2 + 25y^2 = 225$, then find the eccentricity and foci.**

Thinking Process

Find the values of a and b by the given equation of ellipse, then use the formula $b^2 = a^2(1 - e^2)$ to get the value of e.

Sol. Given equation of ellipse, $9x^2 + 25y^2 = 225$

$\Rightarrow \quad \frac{x^2}{25} + \frac{y^2}{9} = 1$

$\Rightarrow \quad a = 5, b = 3$

We know that, $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad 9 = 25(1 - e^2)$

$\Rightarrow \quad \frac{9}{25} = 1 - e^2$

$\Rightarrow \quad e^2 = 1 - 9/25$

$\therefore \quad e = \sqrt{1 - 9/25} = \sqrt{\frac{25 - 9}{25}}$

$= \sqrt{\frac{16}{25}} = 4/5$

Foci $= (\pm\, ae, 0) = (\pm\, 5 \times 4/5, 0) = (\pm\, 4, 0)$

Q. 13 If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find latusrectum of the ellipse.

Sol. Given that, eccentricity $= \frac{5}{8}$, *i.e.*, $e = \frac{5}{8}$

Let equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Since the foci of this ellipse is $(\pm ae, 0)$.

$\therefore$ Distance between foci $= \sqrt{(ae + ae)^2}$

$\Rightarrow \quad 2\sqrt{a^2e^2} = 10$ [$\because$ distance between its foci = 10]

$\Rightarrow \quad \sqrt{a^2e^2} = 5$

$\Rightarrow \quad a^2e^2 = 25$

$\Rightarrow \quad a^2 = \frac{25 \times 64}{25}$

$\therefore \quad a = 8$

We know that,

$\Rightarrow \quad b^2 = a^2(1 - e^2)$

$\Rightarrow \quad b^2 = 64\left(1 - \frac{25}{64}\right)$

$\Rightarrow \quad b^2 = 64\left(\frac{64 - 25}{64}\right)$

$b^2 = 39$

$\therefore$ Length of latusrectum of ellipse $= \frac{2b^2}{a} = 2\left(\frac{39}{8}\right) = \frac{39}{4}$

Q. 14 Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, latusrectum is 5 and the centre is (0, 0).

Thinking Process

First of all find the values of a and b using the formula $b^2 = a^2(1 - e^2)$, then get the equation of the ellipse.

Sol. Given that, $e = 2/3$ and latusrectum = 5

i.e., $\frac{2b^2}{a} = 5 \Rightarrow b^2 = \frac{5a}{2}$

We know that, $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad \frac{5a}{2} = a^2\left(1 - \frac{4}{9}\right)$

$\Rightarrow \quad \frac{5}{2} = \frac{5a}{9} \Rightarrow a = 9/2 \Rightarrow a^2 = \frac{81}{4}$

$\Rightarrow \quad b^2 = \frac{5 \times 9}{2 \times 2} = \frac{45}{4}$

So, the required equation of the ellipse is $\frac{4x^2}{81} + \frac{4y^2}{45} = 1$.

Q. 15 Find the distance between the directrices of ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1.$

Sol. The equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{20} = 1.$

On comparing this equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a = 6, b = 2\sqrt{5}$$

We know that, $b^2 = a^2 (1 - e^2)$

$\Rightarrow$ $20 = 36 (1 - e^2)$

$\Rightarrow$ $\frac{20}{36} = 1 - e^2$

$\therefore$ $e = \sqrt{1 - \frac{20}{36}} = \sqrt{\frac{16}{36}}$

$$E = \frac{4}{6} = \frac{2}{3}$$

Now, directrices $= \left(+\frac{a}{e}, -a/e\right)$

$\therefore$ $\frac{a}{e} = \frac{6}{\frac{2}{3}} = \frac{6 \times 3}{2} = 9$

and $-\frac{a}{e} = -9$

$\therefore$ Distance between the directrices $= |9 - (-9)| = 18$

Q. 16 Find the coordinates of a point on the parabola $y^2 = 8x$, whose focal distance is 4.

Thinking Process

The distance of a point (h, k) from the focus S is called the focal distance of the point P. The focal distance of any point P (h, k) on the parabola $y^2 = 4ax$ is $|h + a|$.

Sol. Given parabola is $y^2 = 8x$ ... (i)

On comparing this parabola to the $y^2 = 4ax$, we get

$$8x = 4ax \Rightarrow a = 2$$

$\therefore$ Focal distance $= |x + a| = 4$

$\Rightarrow$ $|x + 2| = 4$

$\Rightarrow$ $x + 2 = \pm 4$

$\Rightarrow$ $x = 2, -6$

But $x \neq -6$

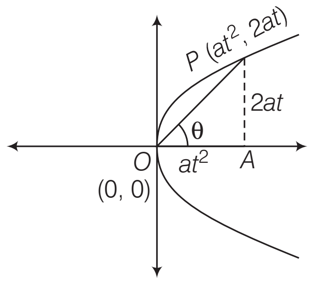
For $x = 2$, $y^2 = 8 \times 2$

$\therefore$ $y^2 = 16 \Rightarrow y = \pm 4$

So, the points are (2, 4) and (2, – 4).

Q. 17 Find the length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola, where the line segment makes an angle θ to the X-axis.

Sol. Given equation of the parabola is $y^2 = 4ax$



Let the coordinates of any point P on the parabola be $(at^2, 2at)$.

In ΔPOA, $\tan\theta = \dfrac{2at}{at^2} = \dfrac{2}{t}$

$\Rightarrow$ $\tan\theta = \dfrac{2}{t} \Rightarrow t = 2\cot\theta$

$\therefore$ length of $OP = \sqrt{(0 - at^2)^2 + (0 - 2at)^2}$

$$= \sqrt{a^2t^4 + 4a^2t^2}$$

$$= at\sqrt{t^2 + 4}$$

$$= 2a\cot\theta\sqrt{4\cot^2\theta + 4}$$

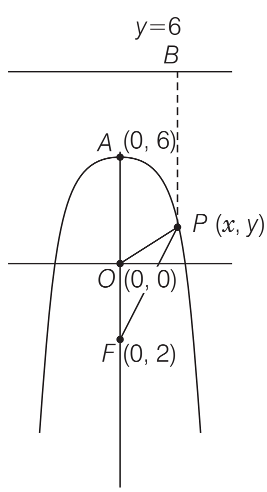
$$= 4a\cot\theta\sqrt{1 + \cot^2\theta}$$

$$= 4a\cot\theta \cdot \operatorname{cosec}\theta$$

$$= \frac{4a\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} = \frac{4a\cos\theta}{\sin^2\theta}$$

Q. 18 If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola.

Sol. Given that the coordinates, vertex of the parabola (0, 4) and focus of the parabola (0, 2).



By definition of the parabola, $PB = PF$

$$\Rightarrow \quad \left|\frac{0 + y - 6}{\sqrt{0+1}}\right| = \sqrt{(x-0)^2 + (y-2)^2}$$

$$\Rightarrow \quad |y - 6| = \sqrt{x^2 + y^2 - 4y + 4}$$

$$\Rightarrow \quad x^2 + y^2 - 4y + 4 = y^2 + 36 - 12y$$

$$\Rightarrow \quad x^2 + 8y = 32$$

Q. 19 If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$, then find the value of m.

Sol. Given that, line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$.

$$\therefore \quad y = mx + 1 \quad \text{...(i)}$$

and

$$y^2 = 4x \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$m^2x^2 + 2mx + 1 = 4x$$

$$\Rightarrow \quad m^2x^2 + 2mx - 4x + 1 = 0$$

$$\Rightarrow \quad m^2x^2 + x(2m - 4) + 1 = 0$$

$$\Rightarrow \quad (2m-4)^2 - 4m^2 \times 1 = 0$$

$$\Rightarrow \quad 4m^2 + 16 - 16m - 4m^2 = 0$$

$$\Rightarrow \quad 16m = 16$$

$$\therefore \quad m = 1$$

Q. 20 If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of the hyperbola.

Thinking Process

First of all find the value of a and b using the given condition, then put them in $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ *to get the required equation of the hyperbola.*

Sol. Distance between the foci *i.e.*, $2ae = 16 \Rightarrow ae = 8$

and $e = \sqrt{2}$

$$\therefore \quad a\sqrt{2} = 8$$

$$a = 4\sqrt{2}$$

We know that, $b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad b^2 = (4\sqrt{2})^2[(\sqrt{2})^2 - 1]$$

$$= 16 \times 2\,(2 - 1)$$

$$= 32\,(2 - 1)$$

So, the equation of hyperbola is

$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$\Rightarrow \quad x^2 - y^2 = 32$$

Q. 21 Find the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$.

Sol. Given equation of the hyperbola is

$$9y^2 - 4x^2 = 36$$

$$\Rightarrow \quad \frac{9y^2}{36} - \frac{4x^2}{36} = \frac{36}{36}$$

$$\Rightarrow \quad \frac{y^2}{4} - \frac{x^2}{9} = 1$$

$$\Rightarrow \quad -\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Since, this equation in form of $-\frac{x^2}{a} + \frac{y^2}{b^2} = 1$, where $a = 3$ and $b = 2$.

$$\therefore \quad e = \sqrt{1 + \left(\frac{a}{b}\right)^2}$$

$$= \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

Q. 22 Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$.

Sol. Given that eccentricity *i.e.*, $e = 3/2$ and $(\pm ae, 0) = (\pm 2, 0)$

$$\therefore \quad ae = 2$$

$$\Rightarrow \quad a \cdot \frac{3}{2} = 2 \Rightarrow a = 4/3$$

$$\because \quad b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow \quad b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right)$$

$$\Rightarrow \quad b^2 = \frac{16}{4}\left(\frac{5}{4}\right) = +\frac{20}{9}$$

So, the equation of hyperbola is

$$\frac{x^2}{\frac{16}{9}} - \frac{y^2}{\frac{20}{9}} = 1$$

$$\Rightarrow \quad \frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$$

Long Answer Type Questions

Q. 23 If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Thinking Process

First of all find the intersection point of the given lines, then get radius of circle from given area. Now, use formula equation of circle with centre (h, k) and radius a is $(x-h)^2 + (y-k)^2 = a^2$.

Sol. Given lines are $2x - 3y - 5 = 0$ ... (i)

and $3x - 4y - 7 = 0$

From Eqs. (i) and (ii), $\frac{x}{21-20} = \frac{y}{-15+14} = \frac{1}{-8+9}$

$\Rightarrow \frac{x}{1} = \frac{y}{-1} = \frac{1}{+1}$

$\Rightarrow x = \pm 1, y = -1$

Since the intersection point of these lines will be coordinates of the circle *i.e.*, coordinates of the circle as (1, – 1).

Let the radius of the circle is r.

Then $\pi r^2 = 154$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22}$

$\Rightarrow r^2 = \frac{14 \times 7}{2} \Rightarrow r^2 = 49$

So, the equation of circle is

$$(x-1)^2 + (y+1)^2 = 49$$

$\Rightarrow x^2 - 2x + 1 + y^2 + 2y + 1 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

Q. 24 Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$.

Sol. Let the general equation of the circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad ...(i)$$

Since, this circle passes through the points (2, 3) and (4, 5).

$\therefore 4 + 9 + 4g + 6f + c = 0$

$\Rightarrow 4g + 6f + c = -13$ ...(ii)

and $16 + 25 + 8g + 10f + c = 0$

$\Rightarrow 8g + 10f + c = -41$ ...(iii)

Since, the centre of the circle $(-g, -f)$ lies on the straight line $y - 4x + 3 = 0$

i.e., $+4g - f + 3 = 0$ ...(iv)

From Eq. (iv), $4g = f - 3$

On putting $4g = f - 3$ in Eq. (ii), we get

$$f - 3 + 6f + c = -13$$

$\Rightarrow 7f + c = 10$ ... (v)

From Eqs. (ii) and (iii),

$$\begin{array}{r} 8g + 12f + 2c = -26 \\ 8g + 10f + c = -41 \\ - \quad - \quad - \quad\quad + \\ \hline 2f + c = 15 \end{array} \qquad \text{...(vi)}$$

From Eqs. (ii) and (vi),

$$\begin{array}{r} 7f + c = -10 \\ 2f + c = 15 \\ - \quad - \quad - \\ \hline 5f = -25 \end{array}$$

$\therefore$ $f = -5$

Now, $c = 10 + 15 = 25$

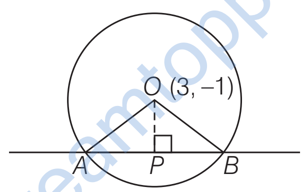
From Eq. (iv), $4g + 5 + 3 = 0$

$\Rightarrow$ $g = -2$

From Eq. (i), equation of the circle is $x^2 + y^2 - 4x - 10y + 25 = 0$.

Q. 25 **Find the equation of a circle whose centre is (3, – 1) and which cuts off a chord 6 length 6 units on the line $2x - 5y + 18 = 0$.**

Sol. Given centre of the circle is (3, – 1).



Now, $OP = \left|\dfrac{6 + 5 + 18}{\sqrt{4 + 25}}\right| = \dfrac{29}{\sqrt{29}} = \sqrt{29}$

In Δ *OPB*, $OB^2 = OP^2 + PB^2$ $\quad [\because AB = 6 \Rightarrow PB = 3]$

$\Rightarrow$ $OB^2 = 29 + 9 \Rightarrow OB^2 = 38$

So, the radius of circle is $\sqrt{38}$,

$\therefore$ Equation of the circle with radius $r = \sqrt{38}$ and centre (3,–1) is

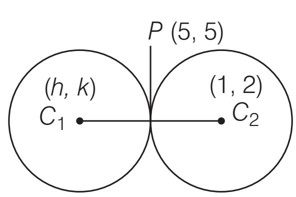
$\Rightarrow$ $(x - 3)^2 + (y + 1)^2 = 38$

$\Rightarrow$ $x^2 - 6x + 9 + y^2 + 2y + 1 = 38$

$x^2 + y^2 - 6x + 2y = 28$

Q. 26 **Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).**

Sol. Let the coordinates of centre of the required circle are (*h*, *k*), then the centre of another circle is (1, 2).



Radius $= \sqrt{1 + 4 + 20} = 5$

So, it is clear that P is the mid-point of C_1C_2.

$\therefore \quad 5 = \dfrac{1+h}{2} \Rightarrow h = 9$

and $\quad 5 = \dfrac{2+k}{2} \Rightarrow k = 8$

So, the equation of and required circle is

$$(x-9)^2 + (y-8)^2 = 25$$

$\Rightarrow \quad x^2 - 18x + 81 + y^2 - 16y + 64 = 25$

$\Rightarrow \quad x^2 + y^2 - 18x - 16y + 120 = 0$

Q. 27 Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line $y = x - 1$.

Thinking Process

First of all let the equation of a circle with centre (h, k) and radius r is $(x-h)^2 + (y-h)^2 = r^2$, then we get the value of (h, k) using given condition.

Sol. Let equation of circle be

$$(x-h)^2 + (y-k)^2 = r^2$$

$\Rightarrow \quad (x-h)^2 + (y-k)^2 = 9 \quad$ ...(i)

Given that, centre (h, k) lies on the line

$y = x - 1$ *i.e.*, $k = h - 1 \quad$ ...(ii)

Now, the circle passes through the point (7, 3).

$\therefore \quad (7-h)^2 + (3-k)^3 = 9$

$\Rightarrow \quad 49 - 14h + h^2 + 9 - 6k + k^2 = 9$

$\Rightarrow \quad h^2 + k^2 - 14h - 6k + 49 = 0 \quad$ ...(iii)

On putting $k = h - 1$ in Eq. (iii), we get

$$h^2 + (h-1)^2 - 14h - 6(h-1) + 49 = 0$$

$\Rightarrow \quad h^2 + h^2 - 2h + 1 - 14h - 6h + 6 + 49 = 0$

$\Rightarrow \quad 2h^2 - 22h + 56 = 0$

$\Rightarrow \quad h^2 - 11h + 28 = 0$

$\Rightarrow \quad h^2 - 7h - 4h + 28 = 0$

$\Rightarrow \quad h(h-7) - 4(h-7) = 0$

$\Rightarrow \quad (h-7)(h-4) = 0$

$\therefore \quad h = 4, 7$

When $\quad h = 7$, then $k = 7 - 1 = 6$

$\therefore$ Centre (7, 6)

When $\quad h = 4$, then $k = 3$

$\therefore$ Centre – (4, 3)

So, the equation of circle when centre (7, 6), is

$$(x-7)^2 + (y-6)^2 = 9$$

$\Rightarrow \quad x^2 - 14x + 49 + y^2 - 12y + 36 = 9$

$\Rightarrow \quad x^2 + y^2 - 14x - 12y + 76 = 0$

When centre (4, 3), then the equation of the circle is

$$(x-4)^2 + (y-3)^2 = 9$$

$\Rightarrow \quad x^2 - 8x + 16 + y^2 - 6y + 9 = 9$

$\Rightarrow \quad x^2 + y^2 - 8x - 6y + 16 = 0$

Q. 28 Find the equation of each of the following parabolas

(i) directrix = 0, focus at (6, 0)

(ii) vertex at (0, 4), focus at (0, 2)

(iii) focus at (– 1, – 2), directrix $x - 2y + 3 = 0$

Sol. **(i)** Given that, directrix = 0 and focus = (6, 0)

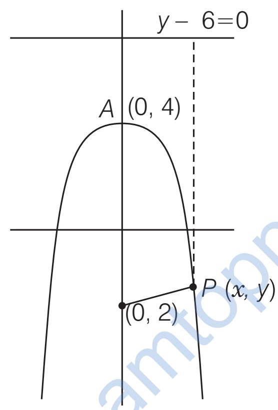
So, the equation of the parabola

$$(x - 6)^2 + y^2 = x^2$$

$$\Rightarrow \quad x^2 + 36 - 12x + y^2 = x^2$$

$$\Rightarrow \quad y^2 - 12x + 36 = 0$$

(ii) Given that, vertex = (0, 4) and focus = (0, 2)



So, the equation of parabola is

$$\sqrt{(x - 0)^2 + (y - 2)^2} = |y - 6|$$

$$\Rightarrow \quad x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$$

$$\Rightarrow \quad x^2 - 4y + 12y - 32 = 0$$

$$\Rightarrow \quad x^2 + 8y - 32 = 0$$

$$\Rightarrow \quad x^2 = 32 - 8y$$

(iii) Given that, focus at (– 1, – 2) and directrix $x - 2y + 3 = 0$

So, the equation of parabola is $\sqrt{(x + 1)^2 + (y + 2)^2} = \left|\dfrac{x - 2y + 3}{\sqrt{1 + 4}}\right|$

$$\Rightarrow \quad x^2 + 2x + 1 + y^2 + 4y + 4 = \frac{1}{5}[x^2 + 4y^2 + 9 - 4xy - 12y + 6x]$$

$$\Rightarrow \quad 4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$$

Q. 29 Find the equation of the set of all points the sum of whose distances from the points (3, 0), (9, 0) is 12.

Sol. Let the coordinates of the point be (x, y), then according to the question,

$$\sqrt{(x - 3)^2 + y^2} + \sqrt{(x - 9)^2 + y^2} = 12$$

$$\Rightarrow \quad \sqrt{(x - 3)^2 + y^2} = 12 - \sqrt{(x - 9)^2 + y^2}$$

On squaring both sides, we get

$$x^2 - 6x + 9 + y^2 = 144 + (x^2 - 18x + 81 + y^2) - 24\sqrt{(x-9)^2 + y^2}$$

$$\Rightarrow \quad 12x - 216 = -24\sqrt{(x-9)^2 + y^2}$$

$$\Rightarrow \quad x - 18 = -2\sqrt{(x-9)^2 + y^2}$$

$$\Rightarrow \quad x^2 - 36x + 324 = 4(x^2 - 18x + 81 + y^2)$$

$$\Rightarrow \quad 3x^2 + 4y^2 - 36x = 0$$

Q. 30 Find the equation of the set of all points whose distance from (0, 4) are $\frac{2}{3}$ of their distance from the line $y = 9$.

Thinking Process

Consider the points (x, y), and apply the condition given in the problem, then get the set of all points.

Sol. Let the point be $P(x, y)$.

$\therefore$ Distance from (0, 4) $= \sqrt{x^2 + (y-4)^2}$

So, the distance from the line $y = 9$ is $\left|\frac{y-9}{\sqrt{1}}\right|$

$$\therefore \quad \sqrt{x^2 + (y-4)^2} = \frac{2}{3}\left|\frac{y-9}{1}\right|$$

$$\Rightarrow \quad x^2 + y^2 - 8y + 10 = \frac{4}{9}(y^2 - 18y + 81)$$

$$\Rightarrow \quad 9x^2 + 9y^2 - 72y + 144 = 4y^2 - 72y + 324$$

$$\Rightarrow \quad 9x^2 + 5y^2 = 180$$

Q. 31 Show that the set of all points such that the difference of their distances from (4, 0) and (– 4, 0) is always equal to 2 represent a hyperbola.

Sol. Let the points be $P(x, y)$.

$\therefore$ Distance of P from (4, 0) $\sqrt{(x-4)^2 + y^2}$ ... (i)

and the distance of P from (– 4, 0) $\sqrt{(x+4)^2 + y^2}$ ... (ii)

Now, $$\sqrt{(x+4)^2 + y^2} - \sqrt{(x-4)^2 + y^2} = 2$$

$$\Rightarrow \quad \sqrt{(x+4)^2 + y^2} = 2 + \sqrt{(x-4)^2 + y^2}$$

On squaring both sides, we get

$$x^2 + 8x + 16 + y^2 = 4 + x^2 - 8x + 16 + y^2 + 4\sqrt{(x-4)^2 + y^2}$$

$$\Rightarrow \quad 16x - 4 = 4\sqrt{(x-4)^2 + y^2}$$

$$\Rightarrow \quad 4(4x - 1) = 4\sqrt{(x-4)^2 + y^2}$$

$$\Rightarrow \quad 16x^2 - 8x + 1 = x^2 + 16 - 8x + y^2$$

$$\Rightarrow \quad 15x^2 - y^2 = 15 \text{ which is a parabola.}$$

Q. 32 Find the equation of the hyperbola with

(i) Vertices $(\pm 5, 0)$, foci $(\pm 7, 0)$

(ii) Vertices $(0, \pm 7)$, $e = \frac{7}{3}$.

(iii) Foci $(0, \pm \sqrt{10})$, passing through $(2, 3)$.

Sol. **(i)** Given that, vertices $= (\pm 5, 0)$, foci $= (\pm 7, 0)$ and $a = \pm 5$

$\therefore \quad (\pm ae, 0) = (\pm 7, 0)$

Now $\quad ae = 7 \Rightarrow 5e = 7$

$\Rightarrow \quad e = 7/5$

$\because \quad b^2 = a^2 (e^2 - 1)$

$\Rightarrow \quad b^2 = 25\left(\frac{49}{25} - 1\right)$

$\Rightarrow \quad b^2 = 25\left(\frac{49 - 25}{25}\right)$

$\Rightarrow \quad b^2 = 24$

So,the equation of parabola is

$$\frac{x^2}{25} - \frac{y^2}{24} = 1 \qquad [\because a^2 = 25 \text{ and } b^2 = 24]$$

(ii) Vertices $= (0, \pm 7)$, $e = 4/3$

$\therefore \quad b = 7, e = 4/3$

$\because \quad e^2 = 1 + \frac{a^2}{b^2}$

$\Rightarrow \quad \frac{16}{9} - 1 = \frac{a^2}{49}$

$\Rightarrow \quad \frac{7}{9} = \frac{a^2}{49} \Rightarrow a^2 = \frac{343}{9}$

So, the equation of hyperbola is

$$-\frac{x^2 \times 9}{343} + \frac{y^2}{49} = 1$$

$\Rightarrow \quad -\frac{9x^2}{7} + y^2 = 49$

$\Rightarrow \quad 9x^2 - 7y^2 + 343 = 0$

(iii) Given that, foci $= (0, \pm \sqrt{10})$

$\because \quad be = \sqrt{10}$

$\Rightarrow \quad a^2 + b^2 = 10$

$\Rightarrow \quad a^2 = 10 - b^2$

$\therefore$ Equation of the hyperbola be

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{...(i)}$$

Since, this hyperbola passes through the point (2, 3).

$\therefore \quad -\frac{4}{a^2} + \frac{9}{b^2} = 1$

$\Rightarrow \quad \frac{-4}{10 - b^2} + \frac{9}{b^2} = 1$

$$\Rightarrow \quad \frac{-4b^2 + 90 - 9b^2}{b^2(10 - b^2)} = 1$$

$$\Rightarrow \quad -13b^2 + 90 = 10b^2 + b^4$$

$$\Rightarrow \quad b^4 - 23b^2 + 90 = 0$$

$$\Rightarrow \quad b^4 - 18b^2 - 5b^2 + 90 = 0$$

$$\Rightarrow \quad b^2(b^2 - 18) - 5(b^2 - 18) = 0$$

$$\Rightarrow \quad (b^2 - 18)(b^2 - 5) = 0$$

$$\Rightarrow \quad b^2 = 18 \Rightarrow b = \pm 3\sqrt{2}$$

or $\quad b^2 = 5 \Rightarrow b = \sqrt{5}$

$\therefore \quad b^2 = 18$ then $a^2 = -8$ [not possible]

When $\quad a^2 = 5$, then $b^2 = 5$

So, the equation of hyperbola is

$$-\frac{x^2}{5} + \frac{y^2}{5} = 1$$

$$\Rightarrow \quad y^2 - x^2 = 5$$

True/False

Q. 33 The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

Thinking Process

If a line is the diameter of circle, then the centre of the circle should lies on line. Use this property to solve the given problem.

Sol. ***False***

Given equation of the circle is

$$x^2 + y^2 + 6x + 2y = 0$$

$\therefore \quad$ Centre $= (-3, -1)$

Since given line is $x + 3y = 0$.

$$\Rightarrow \quad -3 - 3 \neq 0$$

So, this line is not diameter of the circle.

Q. 34 The shortest distance from the point $(2, -7)$ to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is equal to 5.

Sol. ***False***

Given circle is $x^2 + y^2 - 14x - 10y - 151 = 0$.

$\therefore \quad$ Centre $= (7, 5)$

and $\quad$ Radius $= \sqrt{49 + 25 + 151} = \sqrt{225} = 15$

So, the distance between the point $(2, -7)$ and centre of the circle is given by

$$d_1 = \sqrt{(2-7)^2 + (-7-5)^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13$$

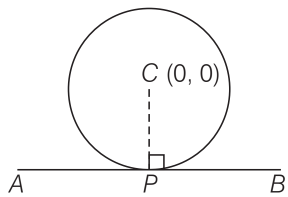
$\therefore$ Shortest distance, $d = |13 - 15| = 2$

Q. 35 If the line $lx + my = 1$ is a tangent to the circle $x^2 + y^2 = a^2$, then the point (l, m) lies on a circle.

Sol. ***True***

Given circle is $x^2 + y^2 = a^2$ ...(i)

$\therefore$ Radius of circle $= a$ and centre $= (0, 0)$



$\therefore$ Distance from point (l, m) and centre is $\sqrt{(0-e)^2 + (0-m)^2} = a$

$\Rightarrow \quad l^2 + m^2 = a^2$

So, l, m lie on the circle.

Q. 36 The point (1, 2) lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

Thinking Process

If the x_1, y_1 lies inside the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$ and it $S > 0$, then the point lies outside the circle.

Sol. ***False***

Given circle is $S \equiv x^2 + y^2 - 2x + 6y + 1 = 0$.

Since, the point is (1, 2).

Now, $\quad S_1 \equiv 1 + 4 - 2 + 12 + 1$

$\Rightarrow \quad S_1 > 0$

So, the (1, 2) lies outside the circle.

Q. 37 The line $lx + my + n = 0$ will touch the parabola $y^2 = 4ax$, if $\text{In} = am^2$.

Sol. ***True***

Given equation of a line is

$$lx + my + n = 0 \quad \text{...(i)}$$

and $\quad$ parabola $y^2 = 4ax$ ...(ii)

From Eq. (i), $x = -\left(\dfrac{my + n}{l}\right)$ put in Eq. (ii), we get

$$y^2 = -\frac{4a\,(my + n)}{l}$$

$\Rightarrow \quad ly^2 = -4a\,my - 4\,ax$

$\Rightarrow \quad ly^2 + 4\,amy + 4an = 0$

For tangent, $\quad D = 0$

$\Rightarrow \quad 16\,a^2m^2 = 4l \times 4\,an$

$\Rightarrow \quad 16\,a^2m^2 = 16\,anl$

$\Rightarrow \quad am^2 = nl$

Q. 38 If P is a point on the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ whose foci are S and S', then $PS+PS'=8$.

Sol. ***False***

Given equation of the ellipse is $\frac{x}{16}+\frac{y^2}{25}=1.$

which is in form of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b>a$

$\therefore$ Foci, $S=(0, be), S'(0, -be)$

$$\therefore \quad e=\sqrt{1-\frac{a^2}{b^2}}$$

$$=\sqrt{\frac{25-16}{25}}=3/5$$

$$\text{Foci, } S=\left(0, \frac{3\times 5}{5}\right), S'=\left(0, -\frac{3\times 5}{5}\right) \text{ i.e., } S=(0,3), S'=(0,-3)$$

Let the coordinate of point P be (x, y) then $PS+PS'=2b=2\times 5=10$

Q. 39 The line $2x+3y=12$ touches the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=2$ at the point (3,2).

Sol. ***True***

Given equation of line is

$$2x+3y=12 \quad \text{...(i)}$$

and ellipse $\frac{x^2}{9}+\frac{y^2}{4}=2$ ...(ii)

Since, the equation of tangent at (x_1, y_1) is $\frac{xx_1}{9}+\frac{yy_1}{4}=2.$

$\therefore$ Tangent at (3, 2),

$$\frac{3x}{9}+\frac{2y}{4}=2$$

$$\Rightarrow \quad \frac{x}{3}+\frac{y}{2}=2$$

$\Rightarrow$ $2x+3y=12$, which is a given line.

Hence, the statement is true.

Q. 40 The locus of the point of intersection of lines $\sqrt{3}x-y-4\sqrt{3}k=0$ and $\sqrt{3}kx+ky-4\sqrt{3}=0$ for different value of k is a hyperbola whose eccentricity is 2.

Thinking Process

First of all eliminate k from the given equations of line, then get the equation of hyperbola.

Sol. ***True***

Given equations of line are

$$\sqrt{3}x-y-4\sqrt{3}k=0 \quad \text{...(i)}$$

and $\sqrt{3}kx+ky-4\sqrt{3}=0$ ...(ii)

From Eq. (i), $4\sqrt{3}\,k=\sqrt{3}\,x-y$

$$\Rightarrow \quad k = \frac{\sqrt{3}x - y}{4\sqrt{3}} \text{ put in Eq. (ii), we get}$$

$$\sqrt{3}x\left(\frac{\sqrt{3}x - y}{4\sqrt{3}}\right) + \left(\frac{\sqrt{3}x - y}{4\sqrt{3}}\right) y - 4\sqrt{3} = 0$$

$$\Rightarrow \quad \frac{1}{4}(\sqrt{3}x^2 - xy) + \frac{1}{4}\left(xy - \frac{y^2}{\sqrt{3}}\right) - 4\sqrt{3} = 0$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4}x^2 - \frac{y^2}{4\sqrt{3}} - 4\sqrt{3} = 0$$

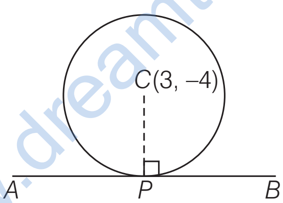
$$\Rightarrow \quad 3x^2 - y^2 - 48 = 0$$

$$\Rightarrow \quad 3x^2 - y^2 = 48, \text{ which is a hyperbola.}$$

Fillers

Q. 41 The equation of the circle having centre at (3, – 4) and touching the line $5x + 12y - 12 = 0$ is ....

Sol. The perpendicular distance from centre (3, – 4) to the line is, $d = \left|\frac{15 - 48 - 12}{\sqrt{25 + 144}}\right| = \frac{45}{13}$



So, the required equations of the circle is $(x - 3)^2 + (y + 4)^2 = \left(\frac{45}{13}\right)^2$.

Q. 42 The equation of the circle circumscribing the triangle whose sides are the lines $y = x + 2$, $3y = 4x$, $2y = 3x$ is ....

Sol. Given equations of line are

$$y = x + 2 \quad \text{...(i)}$$
$$3y = 4x \quad \text{...(ii)}$$
$$2y = 3x \quad \text{...(iii)}$$

From Eqs. (i) and (ii),

$$\frac{4x}{3} = x + 2$$

$$\Rightarrow \quad 4x = 3x + 6$$

$\Rightarrow x = 6$

On putting $x = 6$ in Eq. (i), we get

$$y = 8$$

$$\therefore \quad \text{Point, } A = (6, 8)$$

From Eqs. (i) and (iii),

$$\frac{3x}{2} = x + 2$$

$$\Rightarrow \quad 3x = 2x + 4 \Rightarrow x = 4$$

When $x = 4$, then $y = 6$

$\therefore$ Point, $B = (4, 6)$

From Eqs. (ii) and (iii) $x_1 = 0_1,\ y = 0$

Now, $C = (0, 0)$

Let the equation of circle is

$$x^2 + y^2 + 2g\,xn + 2fy + c = 0$$

Since, the points A (6, 8), B (4, 6) and C (0, 0) lie on this circle.

$$36 + 64 + 12g + 16f + c = 0$$

$$\Rightarrow \quad 12g + 16f + c = -100 \quad \text{...(iv)}$$

and $$16 + 36 + 8g + 12f + c = 0$$

$$\Rightarrow \quad 8g + 12f + c = -52 \quad \text{...(v)}$$

$$\Rightarrow \quad c = 0 \quad \text{...(vi)}$$

From Eqs. (iv), (v) and (vi),

$$12g + 16f = -100$$

$$\Rightarrow \quad 3g + 4f + 25 = 0$$

$$\Rightarrow \quad 2g + 3f + 13 = 0$$

$$\Rightarrow \quad \frac{g}{+52 - 75} = \frac{f}{50 - 39} = \frac{1}{9 - 8}$$

$$\Rightarrow \quad \frac{g}{-23} = \frac{f}{11} = \frac{1}{1}$$

$$\Rightarrow \quad g = -23, f = 11$$

So, the equation of circle is

$$x^2 + y^2 - 46x + 22y + 0 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 46x + 22y = 0$$

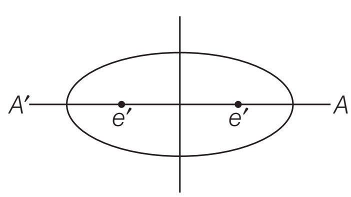
Q. 43 An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 cm and 4 cm, the length of the string and distance between the pins are ....

Sol. Let equation of the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$

$\therefore$ $29 = 6$ and $2b = 4$

$\Rightarrow$ $a = 3$ and $b = 2$

We know that,



$$c^2 = a^2 - b^2 = (3)^2 - (2)^2$$

$$= 9 - 4 = 5 \Rightarrow c = \sqrt{5}$$

$\therefore$ Length of string $= AC' + C'C + AC$

$$= a + c + 2c + ac$$

$$= 2a + 2c = 6 + 2\sqrt{5}$$

$\therefore$ Distances between the pins $= 2\sqrt{5} = cc'$

Q. 44 The equation of the ellipse having foci (0, 1), (0, – 1) and minor axis of length 1 is ……… .

Thinking Process

First of all get the value of a and b with the help of given condition in the problem, then we get the required equation of the ellipse.

Sol. Given that, foci of the ellipse are $(0, \pm be)$.

$\because \quad be \equiv 1$

$\therefore \quad$ Length of minor axis, $2a = 1 \Rightarrow a = 1/2$

$$e^2 = 1 - \frac{a^2}{b^2}$$

$\Rightarrow \quad (be)^2 = b^2 - a^2 \Rightarrow 1 = b^2 - \frac{1}{4}$

$\Rightarrow \quad 1 + \frac{1}{4} = b^2 \Rightarrow \frac{5}{4} = b^2$

So, the equation of ellipse is

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{5/4} = 1 \Rightarrow \frac{4x^2}{1} + \frac{4y^2}{5} = 1$$

Q. 45 The equation of the parabola having focus at (– 1, – 2) and directrix is $x - 2y + 3 = 0$, is ……… .

Sol. Given that, focus at $F(-1, -2)$ and directrix is $x - 2y + 3 = 0$

Let any point on the parabola be (x, y).

$\therefore \quad PF = \left|\frac{x - 2y + 3}{\sqrt{1 + 4}}\right|$

$\Rightarrow \quad (x + 1)^2 + (y + 2)^2 = \frac{(x - 2y + 3)^2}{5}$

$\Rightarrow \quad 5[x^2 + 2x + 1 + y^2 + 4y + 4] = x^2 + 4y^2 + 9 - 4xy - 12y + 6x$

$\Rightarrow \quad 4x^2 + y^2 + 4x + 32y + 16 = 0$

Q. 46 The equation of the hyperbola with vertices at $(0, \pm 6)$ and eccentricity $\frac{5}{3}$ is ……… and its foci are ……… .

Sol. Let the equation of the hyperbola be $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Then vertices $= (0, \pm b) = (0, \pm 6)$

$\therefore \quad b = 6$ and $e = 5/3$

$\because \quad e = \sqrt{1 + \frac{a^2}{b^2}} \Rightarrow \frac{25}{9} = 1 + \frac{a^2}{36}$

$\Rightarrow \quad \frac{25 - 9}{9} = \frac{a^2}{36} \Rightarrow 16 = \frac{a^2}{4} \Rightarrow a^2 = 48$

So, the equation of hyperbola is,

$$\frac{-x^2}{48} + \frac{y^2}{36} = 1 \Rightarrow \frac{y^2}{36} - \frac{x^2}{48} = 1$$

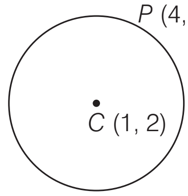
$\because \quad$ Foci $= (0, \pm be) = \left(= 0, \pm \frac{5}{3} \times 6\right) = (0, \pm 10)$

Objective Type Questions

Q. 47 The area of the circle centred at (1, 2) and passing through the point(4, 6) is

(a) 5π (b) 10π
(c) 25π (d) None of these

Sol. *(c)* Given that, centre of the circle is (1, 2).



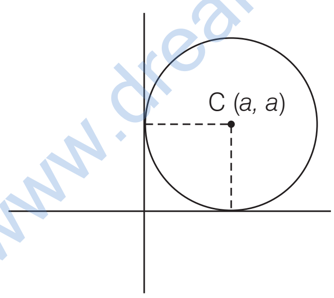
$\because$ $CP = \sqrt{9+16} = 5 =$ Radius of the circle

$\therefore$ Required area $= \pi r^2 = 25\pi$

Q. 48 Equation of a circle which passes through (3, 6) and touches the axes is

(a) $x^2 + y^2 + 6x + 6y + 3 = 0$ (b) $x^2 + y^2 - 6x - 6y - 9 = 0$
(c) $x^2 + y^2 - 6x - 6y + 9 = 0$ (d) None of these

Sol. *(c)* Let centre of the circle be (a, a), then equation of the circle is $(x-a)^2 + (y-a)^2 = a^2$.



Since, the point (3, 6) lies on this circle, then

$$(3-a)^2 + (6-a)^2 = a^2$$

$\Rightarrow$ $a^2 + 9 - 6a + 36 - 12a + a^2 = a^2$

$\Rightarrow$ $a^2 - 18a + 45 = 0$

$\Rightarrow$ $a^2 - 15a - 3a + 45 = 0$

$\Rightarrow$ $a(a-15) - 3(a-15) = 0$

$\Rightarrow$ $(a-3)(a-15) = 0$

$\Rightarrow$ $a = 3, a = 15$

So, the equation of circle is

$$(x-3)^2 + (y-3)^2 = 9$$

$\Rightarrow$ $x^2 - 6x + 9 + y^2 - 6y + 9 = 9$

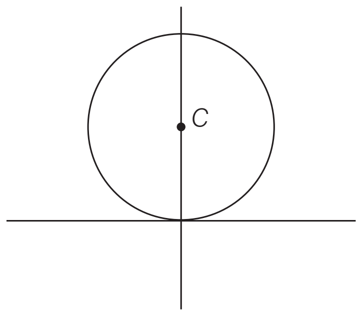
$\Rightarrow$ $x^2 + y^2 - 6x - 6y + 9 = 0$

Q. 49 Equation of the circle with centre on the Y-axis and passing through the origin and the point (2, 3) is

(a) $x^2 + y^2 + 13y = 0$ (b) $3x^2 + 3y^2 + 13x + 3 = 0$

(c) $6x^2 + 6y^2 - 13y = 0$ (d) $x^2 + y^2 + 13x + 3 = 0$

Sol. *(c)* Let general equation of the circle is $x^2 + y^2 + 2gh + 2fy + c = 0$.



Since the point (0, 0) and (2, 3) lie on it $c = 0$.

$\therefore \quad 4 + 9 + 4g + 6f = 0$

$\Rightarrow \quad 2g + 3f = -13/2$

Since the centre lie on Y-axis, then $g = 0$.

$\therefore \quad 3f = -13/2$

$\Rightarrow \quad f = -13/6$

So, the equation of circle is

$$x^2 + y^2 - \frac{13y}{6} = 0$$

$$\Rightarrow \quad 6x^2 + 6y^2 - 13y = 0$$

Q. 50 The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length $3a$ is

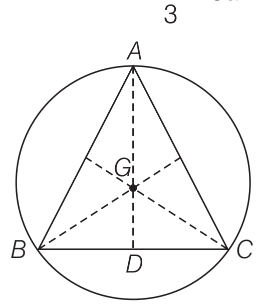
(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

Sol. *(c)* Given that, length of the median $AD = 3a$

$$\because \quad \text{Radius of the circle} = \frac{3}{2} \times \text{Length of median}$$

$$= \frac{2}{3} \times 3a = 2a$$



So, the equation of the circle is $x^2 + y^2 = 4a^2$.

Q. 51 If the focus of a parabola is (0, – 3) and its directrix is $y = 3$, then its equation is

(a) $x^2 = -12y$
(b) $x^2 = 12y$
(c) $y^2 = -12x$
(d) $y^2 = 12x$

Sol. ***(a)*** Given that, focus of parabola at F (0, – 3) and equation of directrix is $y = 3$.

Let any point on the parabola is $P(x, y)$.

Then, $PF = |y - 3|$

$$\Rightarrow \sqrt{(x-0)^2 + (y+3)^2} = |y - 3|$$

$$\Rightarrow x^2 + y^2 + 6y + 9 = y^2 - 6y + 9$$

$$\Rightarrow x^2 + 12y = 0$$

$$\Rightarrow x^2 = -12y$$

Q. 52 If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is

(a) $\frac{2}{3}$
(b) $\frac{4}{3}$
(c) $\frac{1}{3}$
(d) 4

Sol. ***(b)*** Given that, parabola is

$$y^2 = 4ax \quad \text{... (i)}$$

$\therefore$ Length of latusrectum = $4a$

Since, the parabola passes through the point (3, 2).

Then, $4 = 4a(3)$

$$\Rightarrow a = 1/3$$

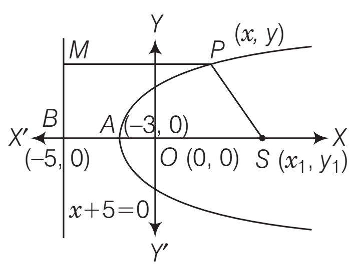
$$\therefore 4a = 4/3$$

Q. 53 If the vertex of the parabola is the point (– 3, 0) and the directrix is the line $x + 5 = 0$, then its equation is

(a) $y^2 = 8(x + 3)$
(b) $x^2 = 8(y + 3)$
(c) $y^2 = -8(x + 3)$
(d) $y^2 = 8(x + 5)$

Sol. ***(a)*** Here, vertex = (– 3, 0)

$\therefore a = -3$ and directrix, $x + 5 = 0$



Since, axis of the parabola is a line perpendicular to directrix and A is the mid-point of AS.

Then, $-3 = \frac{x_1 - 5}{2}$

$\Rightarrow$ $-6 = x_1 - 5 \Rightarrow x_1 = -1,$

$0 = \frac{0 + y_1}{2} \Rightarrow y_1 = 0$

$\therefore$ $S = (-1, 0)$

$\because$ $PM = PS$

$\Rightarrow$ $|x + 5| = \sqrt{(x+1)^2 + y^2}$

$\Rightarrow$ $x^2 + 2x + 1 + y^2 = x^2 + 10x + 25$

$\Rightarrow$ $y^2 = + 8x + 24$

$\Rightarrow$ $y^2 = + 8(x + 3)$

Q. 54 If equation of the ellipse whose focus is (1, – 1), then directrix the line $x - y - 3 = 0$ and eccentricity $\frac{1}{2}$ is

(a) $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

(b) $7x^2 + 2xy + 7y^2 + 7 = 0$

(c) $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$

(d) None of the above

Sol. ***(a)*** Given that, focus of the ellipse is (1, – 1) and the equation of directrix is $x - y - 3 = 0$ and $e = \frac{1}{2}$

Let $P(x, y)$ and $F(1, -1)$.

$\therefore$ $\frac{PF}{\text{Distance of } P \text{ from } (x - y - 3 = 0)} = \frac{1}{2}$

$\Rightarrow$ $\frac{\sqrt{(x-1)^2 + (y+1)^2}}{\frac{|x - y - 3|}{\sqrt{2}}} = \frac{1}{2}$

$\Rightarrow$ $\frac{2[x^2 - 2x + 1 + y^2 + 2y + 1]}{(x - y - 3)^2} = \frac{1}{4}$

$\Rightarrow$ $8x^2 - 16x + 16 + 8y^2 + 16y = x^2 + y^2 + 9 - 2xy + 6y - 6x$

$\Rightarrow$ $7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$

Q. 55 The length of the latusrectum of the ellipse $3x^2 + y^2 = 12$ is

(a) 4 (b) 3 (c) 8 (d) $\frac{4}{\sqrt{3}}$

Thinking Process

First of all find the value of a and b from the given equation, after that get length of latusrectum by using formula $\frac{2a^2}{b}$.

Sol. ***(d)*** Given equation of ellipse is

$3x^2 + y^2 = 12$

$\Rightarrow \quad \dfrac{x^2}{4} + \dfrac{y^2}{12} = 1$

$\therefore \quad a^2 = 4 \Rightarrow a = 2$

and $\quad b^2 = 12 \Rightarrow b = 2\sqrt{3}$

$\because \quad b > a$

$\therefore$ Length of latusrectum $= \dfrac{2 \times 4}{2\sqrt{3}} = \dfrac{4}{\sqrt{3}} = \dfrac{2a^2}{b}$

Q. 56 If e is eccentricity of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (where, $a < b$), then

(a) $b^2 = a^2(1 - e^2)$
(b) $a^2 = b^2(1 - e^2)$
(c) $a^2 = b^2(e^2 - 1)$
(d) $b^2 = a^2(e^2 - 1)$

Sol. ***(b)*** Given that, $\quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \; a < b$

We know that, $\quad e = \sqrt{1 - \dfrac{a^2}{b^2}} \Rightarrow e^2 = \dfrac{(b^2 - a^2)}{b^2}$

$\Rightarrow \quad b^2e^2 = b^2 - a^2$

$\Rightarrow \quad a^2 = b^2(1 - e^2)$

Q. 57 The eccentricity of the hyperbola whose latusrectum is 8 and conjugate axis is equal to half of the distance between the foci is

(a) $\dfrac{4}{3}$
(b) $\dfrac{4}{\sqrt{3}}$
(c) $\dfrac{2}{\sqrt{3}}$
(d) None of these

Sol. ***(c)*** Length of latusrectum of the hyperbola *i.e.,*

$$8 = \frac{2b^2}{a} \Rightarrow b^2 = 4a \quad \text{...(i)}$$

$\therefore$ Distance between the foci $= 2ae$

Since, transverse axis be a and conjugate axis be b.

$\therefore \quad \dfrac{1}{2}(2ae) = 2b$

$\Rightarrow \quad ae = 2b \quad$ ...(ii)

$\Rightarrow \quad b^2 = a^2(e^2 - 1) \quad$ ...(iii)

From Eqs. (i) and (ii),

$$4a = \frac{a^2e^2}{4}$$

$\Rightarrow \quad 16a = a^2e^2$

$\Rightarrow \quad 16 = ae^2 \Rightarrow a = \dfrac{16}{e^2}$

$\because \quad 4a = a^2(e^2 - 1)$

$\Rightarrow \quad \dfrac{4}{a} = e^2 - 1$

$\Rightarrow \quad \dfrac{4e^2}{16} = e^2 - 1$

$$\Rightarrow \quad e^2\left(1-\frac{4}{16}\right)=1$$

$$\Rightarrow \quad e^2\left(\frac{12}{16}\right)=1 \Rightarrow e^2=\left(\frac{16}{12}\right)$$

$$\Rightarrow \quad e^2=\frac{4}{3} \Rightarrow e=\frac{2}{\sqrt{3}}$$

Q. 58 The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(a) $x^2 - y^2 = 32$ (b) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(c) $2x - 3y^2 = 7$ (d) None of these

Thinking Process

The distance between the foci of hyperbola is $2ae$ and $b^2 = a^2(e^2 - 1)$. Use this relation to set the value of a and b.

Sol. ***(a)*** Given that, distance between the foci of hyperbola

i.e., $2ae = 16 \Rightarrow ae = 8$ ...(i)

and $e = \sqrt{2}$ ...(ii)

Now, $\sqrt{2}\,a = 8$

$\Rightarrow \quad a = 4\sqrt{2}$

$\because \quad b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad b^2 = 32(2 - 1)$

$\Rightarrow \quad b^2 = 32$

$\therefore \quad \frac{x^2}{32} - \frac{y^2}{32} = 1$

$\Rightarrow \quad x^2 - y^2 = 32$

Q. 59 Equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at $(\pm 2, 0)$ is

(a) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$ (b) $\frac{x^2}{9} - \frac{y^2}{9} = \frac{4}{9}$

(c) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (d) None of these

Sol. ***(a)*** Given that, eccentricity of the hyperbola, $e = 3/2$

and foci $= (\pm 2, 0), (\pm ae, 0)$

$\because \quad ae = 2$

$\Rightarrow \quad a \times 3/2 = 2 \Rightarrow a = 4/3$

$\because \quad b^2 = a^2(e^2 - 1)$

$$\Rightarrow \quad b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right) \Rightarrow b^2 = \frac{16}{9}\left(\frac{5}{4}\right)$$

$$\Rightarrow \quad b^2 = \frac{20}{9}$$

So, the equation of the hyperbola is

$$\frac{x^2}{\frac{16}{9}} - \frac{y^2}{20/9} = 1 \Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$$