(Chapter 12)(Atoms)
XII

Additional Exercises

Question 12.11:

Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Answer

(a) about the same

The average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model is about the same size as predicted by Rutherford's model. This is because the average angle was taken in both models.

(b) much less

The probability of scattering of α-particles at angles greater than 90° predicted by Thomson's model is much less than that predicted by Rutherford's model.

(c) Scattering is mainly due to single collisions. The chances of a single collision increases linearly with the number of target atoms. Since the number of target atoms increase with an increase in thickness, the collision probability depends linearly on the thickness of the target.

(d) Thomson's model

It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of α−particles by a thin foil. This is because a single collision causes very little deflection in this model. Hence, the observed average scattering angle can be explained only by considering multiple scattering.

Question 12.12:

The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Answer

Radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \quad \dots(1)$$

Where,

$\in_0$ = Permittivity of free space

h = Planck's constant = 6.63×10^{-34} Js

m_e = Mass of an electron = 9.1×10^{-31} kg

e = Charge of an electron = 1.9×10^{-19} C

m_p = Mass of a proton = 1.67×10^{-27} kg

r = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is given as:

$$F_C = \frac{e^2}{4\pi \in_0 r^2} \quad \dots(2)$$

Gravitational force of attraction between an electron and a proton is given as:

$$F_G = \frac{G m_p m_e}{r^2} \quad \dots(3)$$

Where,

G = Gravitational constant = 6.67×10^{-11} N m^2/kg^2

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write:

$\therefore F_G = F_C$

$$\frac{Gm_p m_e}{r^2} = \frac{e^2}{4\pi \in_0 r^2}$$

$$\therefore \frac{e^2}{4\pi \in_0} = Gm_p m_e \qquad \dots (4)$$

Putting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e^2}$$

$$= \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2} \approx 1.21\times10^{29}\ m$$

It is known that the universe is 156 billion light years wide or 1.5×10^{27} m wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

Question 12.13:

Obtain an expression for the frequency of radiation emitted when a hydrogen atom deexcites from level n to level (n−1). For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Answer

It is given that a hydrogen atom de-excites from an upper level (n) to a lower level (n−1).

We have the relation for energy (E_1) of radiation at level n as:

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \quad \text{... (i)}$$

Where,

ν_1 = Frequency of radiation at level n

h = Planck's constant

m = Mass of hydrogen atom

e = Charge on an electron

$\in_0$ = Permittivity of free space

Now, the relation for energy (E_2) of radiation at level (n − 1) is given as:

$$E_2 = h\nu_2 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad \text{... (ii)}$$

Where,

ν_2 = Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation:

$E = E_2 - E_1$ $h\nu = E_2 - E_1$ … (iii)

Where,

ν = Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$\nu = \frac{me^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4 (2n-1)}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \simeq 2n$ and $(n-1) \simeq n$.

$$\therefore \nu = \frac{me^4}{32\pi^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \text{... (iv)}$$

Classical relation of frequency of revolution of an electron is given as:

$$\nu_c = \frac{v}{2\pi r} \quad \ldots (v)$$

Where,

Velocity of the electron in the n^{th} orbit is given as:

$$v = \frac{e^2}{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right) n} \quad \ldots (vi)$$

And, radius of the n^{th} orbit is given as:

$$r = \frac{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \quad \ldots (vii)$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$\nu_c = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \ldots (viii)$$

Hence, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

Question 12.14:

Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10} m).

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Answer

(a) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3 \times 10^8$ m/s

Let us take a quantity involving the given quantities as $\left(\dfrac{e^2}{4\pi \epsilon_0 m_e c^2}\right)$.

Where,

ϵ_0 = Permittivity of free space

And, $\dfrac{1}{4\pi \epsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \epsilon_0} \times \frac{e^2}{m_e c^2}$$

$$= 9\times10^9 \times \frac{(1.6\times10^{-19})^2}{9.1\times10^{-31} \times (3\times10^8)^2}$$

$$= 2.81\times10^{-15} \text{ m}$$

Hence, the numerical value of the taken quantity is much smaller than the typical size of an atom.

(b) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

Let us take a quantity involving the given quantities as $\dfrac{4\pi \epsilon_0 \left(\dfrac{h}{2\pi}\right)^2}{m_e e^2}$.

Where,

ϵ_0 = Permittivity of free space

And, $\frac{1}{4\pi\epsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$4\pi\epsilon_0 \times \frac{\left(\frac{h}{2\pi}\right)^2}{m_e e^2}$$

$$= \frac{1}{9\times10^9} \times \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$

$$= 0.53\times10^{-10} \ m$$

Hence, the value of the quantity taken is of the order of the atomic size.

Question 12.15:

The total energy of an electron in the first excited state of the hydrogen atom is about −3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Answer

(a) Total energy of the electron, E = −3.4 eV

Kinetic energy of the electron is equal to the negative of the total energy.

⇒ K = −E

= − (− 3.4) = +3.4 eV

Hence, the kinetic energy of the electron in the given state is +3.4 eV.

(b) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

⇒ U = −2 K

= − 2 × 3.4 = − 6.8 eV

Hence, the potential energy of the electron in the given state is − 6.8 eV.

(c) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since total energy is the sum of kinetic and potential energies, total energy of the system will also change.

Question 12.16:

If Bohr's quantisation postulate (angular momentum = nh/2π) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Answer

We never speak of quantization of orbits of planets around the Sun because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant (h). The angular momentum of the Earth in its orbit is of the order of 10^{70}h. This leads to a very high value of quantum levels n of the order of 10^{70}. For large values of n, successive energies and angular momenta are relatively very small. Hence, the quantum levels for planetary motion are considered continuous.

Question 12.17:

Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$ orbits around a proton].

Answer

Mass of a negatively charged muon, $m_\mu = 207m_e$

According to Bohr's model,

Bohr radius, $r_e \propto \left(\frac{1}{m_e}\right)$

And, energy of a ground state electronic hydrogen atom, $E_e \propto m_e$.

Also, energy of a ground state *muonic hydrogen atom*, $E_\mu \propto m_\mu$.

We have the value of the first Bohr orbit, $r_e = 0.53\ \mathring{A} = 0.53 \times 10^{-10}$ m

Let r_μ be the radius of muonic hydrogen atom.

At equilibrium, we can write the relation as:

$$m_\mu r_\mu = m_e r_e$$

$$207 m_e \times r_\mu = m_e r_e$$

$$\therefore r_\mu = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13}\ \text{m}$$

Hence, the value of the first Bohr radius of a muonic hydrogen atom is

2.56×10^{-13} m.

We have,

$E_e = -13.6$ eV

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e}$$

$$E_\mu = 207 E_e$$

$$= 207 \times (-13.6) = -2.81\ \text{keV}$$

Hence, the ground state energy of a muonic hydrogen atom is −2.81 keV.