XII

Exercises

Question 7.1:

A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Answer 7.1:

Resistance of the resistor, R = 100 Ω

Supply voltage, V = 220 V

Frequency, ν = 50 Hz

(a) The rms value of current in the circuit is given as

$$I = \frac{V}{R} = \frac{220}{100} = 2.20\ A$$

(b) The net power consumed over a full cycle is given as:

P = VI = 220 × 2.2 = 484 W

Question 7.2:

(a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Answer 7.2:

(a) Peak voltage of the ac supply, V_0 = 300 V

rms voltage is given as:

$$V = \frac{V_o}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.1\ V$$

(b) The rms value of current is given as:

I = 10 A

Now, peak current is given as:

$$I_o = \sqrt{2}I = \sqrt{2} \times 10 = 14.1\ A$$

Question 7.3:

A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Answer 7.3:

Inductance of inductor, L = 44 mH = 44×10^{-3} H

Supply voltage, V = 220 V

Frequency, ν = 50 Hz

Angular frequency, ω = $2\pi\nu$

Inductive reactance, $X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times 44 \times 10^{-3}\ \Omega$

rms value of current is given as:

$$I = \frac{V}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}} = 15.92\ A$$

Hence, the rms value of current in the circuit is 15.92 A.

Question 7.4:

A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Answer 7.4:

Capacitance of capacitor, C = 60 μF = 60×10^{-6} F

Supply voltage, V = 110 V

Frequency, ν = 60 Hz

Angular frequency, ω = $2\pi\nu$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}}\ \Omega$$

rms value of current is given as:

$$I = \frac{V}{X_C} = \frac{220}{2\pi \times 60 \times 60 \times 10^{-6}} = 2.49\ A$$

Hence, the rms value of current is 2.49 A.

Question 7.5:

In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Answer 7.5:

In the inductive circuit,

Rms value of current, I = 15.92 A

Rms value of voltage, V = 220 V

Hence, the net power absorbed can be obtained by the relation,

$$P = VI \cos \Phi$$

Where,

Φ = Phase difference between V and I.

For a pure inductive circuit, the phase difference between alternating voltage and current is 90° i.e., Φ= 90°.

Hence, P = 0 i.e., the net power is zero.

In the capacitive circuit,

rms value of current, I = 2.49 A

rms value of voltage, V = 110 V

Hence, the net power absorbed can be obtained as:

$$P = VI \; Cos \; \Phi$$

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90° i.e., $\Phi = 90°$.

Hence, P = 0 i.e., the net power is zero.

Question 7.6:

Obtain the resonant frequency ωr of a series LCR circuit with L = 2.0 H, C = 32 μF and R = 10 Ω. What is the Q-value of this circuit?

Answer 7.6:

Inductance, L = 2.0 H

Capacitance, C = 32 μF = 32 × 10^{-6} F

Resistance, R = 10 Ω

Resonant frequency is given by the relation,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{1}{8 \times 10^{-3}} = 125 \; rad/s.$$

Now, Q-value of the circuit is given as:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}} = \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

Hence, the Q-Value of this circuit is 25.

Question 7.7:

A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Answer 7.7:

Capacitance, C = 30μF = 30×10^{-6} F Inductance, L = 27 mH = 27×10^{-3} H

Angular frequency is given as:

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{9 \times 10^{-4}} = 1.11 \times 10^3 \ rad/s$$

Hence, the angular frequency of free oscillations of the circuit is 1.11×10^3 rad/s.

Question 7.8:

Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

Answer 7.8:

Capacitance of the capacitor, C = 30 μF = 30×10^{-6} F

Inductance of the inductor, L = 27 mH = 27 × 10^{-3} H

Charge on the capacitor, Q = 6 mC = 6 × 10^{-3} C

Total energy stored in the capacitor can be calculated as:

$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}} = \frac{6}{10} = 0.6\,J$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

Question 7.9:

A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Answer 7.9:

At resonance, the frequency of the supply power equals the natural frequency of the given LCR circuit.

Resistance, R = 20 Ω

Inductance, L = 1.5 H

Capacitance, C = 35 μF = 30 × 10^{-6} F

AC supply voltage to the LCR circuit, V = 200 V

Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{At resonance,} \qquad X_L = X_C$$

$$\therefore Z = R = 20\ \Omega$$

Current in the circuit can be calculated as:

$$I = \frac{V}{Z} = \frac{200}{20} = 10\ A$$

Hence, the average power transferred to the circuit in one complete cycle:

$$VI = 200 \times 10 = 2000\ W.$$

Question 7.10:

A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radio wave.]

Answer 7.10:

The range of frequency (ν) of a radio is 800 kHz to 1200 kHz.

Lower tuning frequency, ν_1 = 800 kHz = 800×10^3 Hz

Upper tuning frequency, ν_2 = 1200 kHz = 1200×10^3 Hz

Effective inductance of circuit L = 200 μH = 200×10^{-6} H

Capacitance of variable capacitor for ν_1 is given as:

$$C_1 = \frac{1}{\omega_1^2 L}$$

Where,

ω_1 = Angular frequency for capacitor C_1

$= 2\pi\nu_1$

$= 2\pi \times 800 \times 10^3\ rad/s$

$$\therefore\ C_1 = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200\ \times\ 10^{-6}}$$

$$= 1.9809 \times 10^{-10}\ F\ = 198\ pF$$

Capacitance of variable capacitor for ν_2 is given as:

$$C_2 = \frac{1}{\omega_2^2 L}$$

Where,

ω_2 = Angular frequency for capacitor C_2

$= 2\pi\nu_2$

$= 2\pi \times 1200 \times 10^3\ rad/s$

$$\therefore\ C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200\ \times\ 10^{-6}}$$

$$= 0.8804 \times 10^{-10}\ F\ = 88\ pF$$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

Question 7.11:

Figure 7.21 shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80μF, R = 40 Ω

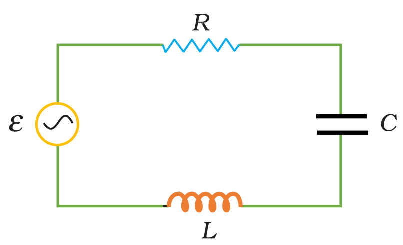

FIGURE 7.21

(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Answer 7.11:

Inductance of the inductor, L = 5.0 H

Capacitance of the capacitor, C = 80 μH = 80 × 10^{-6} F

Resistance of the resistor, R = 40 Ω

Potential of the variable voltage source, V = 230 V

(a) Resonance angular frequency is given as:

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{10^3}{20} = 50\, rad/s$$

Hence, the circuit will come in resonance for a source frequency of 50 rad/s.

(b) Impedance of the circuit is given by the relation:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C \Rightarrow Z = R = 40\ \Omega$

Amplitude of the current at the resonating frequency is given as: $I_o = \frac{V_o}{Z}$

Where,

V_o = Peak voltage = $\sqrt{2}\,V$

$$\therefore I_o = \frac{\sqrt{2}\,V}{Z} = \frac{\sqrt{2}\times 230}{40} = 8.13\,A$$

Hence, at resonance, the impedance of the circuit is 40 Ω and the amplitude of the current is 8.13 A.

(c) rms potential drop across the inductor,

$$(V_L)_{rms} = I \times \omega_r L$$

Where,

$$I_{\text{rms}} = \frac{I_o}{\sqrt{2}} = \frac{\sqrt{2}\,V}{\sqrt{2}\,Z} = \frac{230}{40} = \frac{23}{4}\ A$$

$$\therefore (V_L)_{\text{rms}} = \frac{23}{4} \times 50 \times 5 = 1437.5\,V$$

Potential drop across the capacitor:

$$\therefore (V_C)_{\text{rms}} = I \times \frac{1}{\omega_r C} = \frac{23}{4} \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5\,V$$

Potential drop across the resistor:

$$(V_R)_{\text{rms}} = IR = \frac{23}{4} \times 40 = 230\,V$$

Potential drop across the LC combination:

$$V_{LC} = I(X_L - X_C)$$

At resonance, $X_L = X_C \Rightarrow V_{LC} = 0$

Hence, it is proved that the potential drop across the LC combination is zero at resonating frequency.