

Activity 21

OBJECTIVE

To verify that angle in a semi-circle is a right angle, using vector method.

MATERIAL REQUIRED

Cardboard, white paper, adhesive, pens, geometry box, eraser, wires, paper arrow heads.

METHOD OF CONSTRUCTION

1. Take a thick cardboard of size 30 cm \times 30 cm.
2. On the cardboard, paste a white paper of the same size using an adhesive.
3. On this paper draw a circle, with centre O and radius 10 cm.

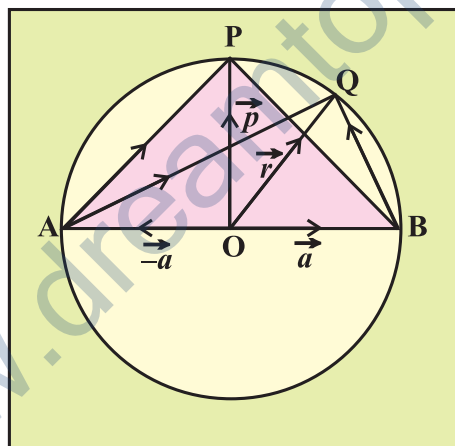


Fig. 21

4. Fix nails at the points O, A, B, P and Q. Join OP, OA, OB, AP, AQ, BQ, OQ and BP using wires.
5. Put arrows on OA, OB, OP, AP, BP, OQ, AQ and BQ to show them as vectors, using paper arrow heads, as shown in the figure.

DEMONSTRATION

1. Using a protractor, measure the angle between the vectors \overrightarrow{AP} and \overrightarrow{BP} , i.e., $\angle APB = 90^\circ$.

- Similarly, the angle between the vectors \overrightarrow{AQ} and \overrightarrow{BQ} , i.e., $\angle AQB = 90^\circ$.
- Repeat the above process by taking some more points R, S, T, ... on the semi-circles, forming vectors AR, BR; AS, BS; AT, BT; ..., etc., i.e., angle formed between two vectors in a semi-circle is a right angle.

OBSERVATION

By actual measurement.

$$|\overrightarrow{OP}| = |\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OQ}| = r = a = p = \underline{\hspace{2cm}},$$

$$|\overrightarrow{AP}| = \underline{\hspace{2cm}}, \quad |\overrightarrow{BP}| = \underline{\hspace{2cm}}, \quad |\overrightarrow{AB}| = \underline{\hspace{2cm}}$$

$$|\overrightarrow{AQ}| = \underline{\hspace{2cm}}, \quad |\overrightarrow{BQ}| = \underline{\hspace{2cm}}$$

$$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2 = \underline{\hspace{2cm}}, \quad |\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2 = \underline{\hspace{2cm}}$$

So, $\angle APB = \underline{\hspace{2cm}}$ and $\overrightarrow{AP} \cdot \overrightarrow{BP} = \underline{\hspace{2cm}}$ $\angle AQB = \underline{\hspace{2cm}}$ and

$$\overrightarrow{AQ} \cdot \overrightarrow{BP} = \underline{\hspace{2cm}}$$

Similarly, for points R, S, T, $\underline{\hspace{2cm}}$

$$\angle ARB = \underline{\hspace{2cm}}, \quad \angle ASB = \underline{\hspace{2cm}}, \quad \angle ATB = \underline{\hspace{2cm}}, \quad \underline{\hspace{2cm}}$$

i.e., angle in a semi-circle is a right angle.

APPLICATION

This activity can be used to explain the concepts of

- opposite vectors
- vectors of equal magnitude

- (iii) perpendicular vectors
- (iv) Dot product of two vectors.

NOTE

Let $OA = OB = a = OP = p$

$$\overrightarrow{OA} = -\vec{a}, \overrightarrow{OB} = \vec{a}, \overrightarrow{OP} = \vec{p}$$

$$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}, \overrightarrow{BP} = \vec{p} - \vec{a}$$

$$\overrightarrow{AP} \cdot \overrightarrow{BP} = (\vec{p} + \vec{a}) \cdot (\vec{p} - \vec{a}) = |\vec{p}|^2 - |\vec{a}|^2 = 0$$

$$\left(\text{since } |\vec{p}|^2 = |\vec{a}|^2 \right)$$

So, the angle APB between the vectors \overrightarrow{AP} and \overrightarrow{BP} is a right angle.

Similarly, $\overrightarrow{AQ} \cdot \overrightarrow{BQ} = 0$, so, $\angle AQB = 90^\circ$ and so on.

Activity 22

OBJECTIVE

To locate the points to given coordinates in space, measure the distance between two points in space and then to verify the distance using distance formula.

METHOD OF CONSTRUCTION

MATERIAL REQUIRED

Drawing board, geometry box, squared paper, nails of different lengths, paper arrows.

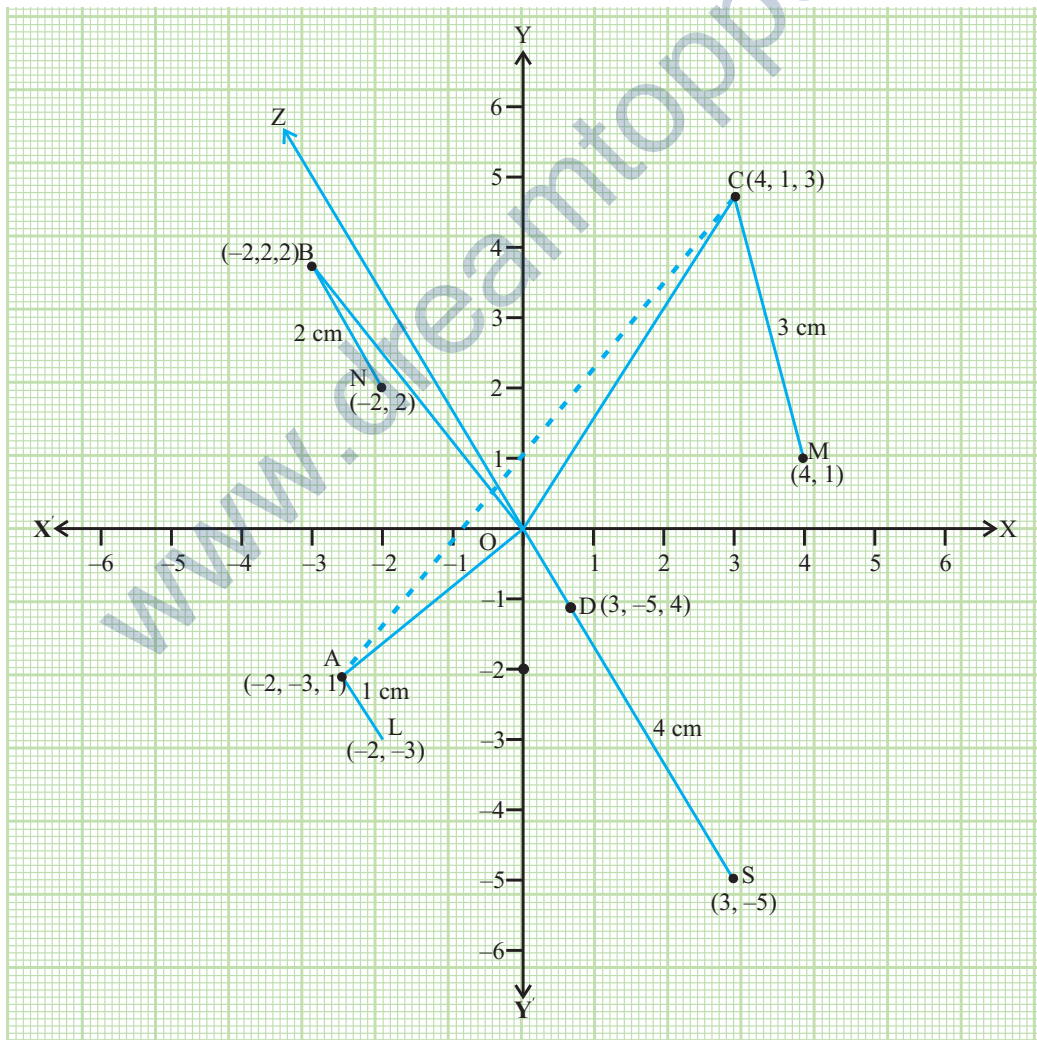


Fig 22

1. Take a drawing board and paste a squared paper on it.
2. Draw two lines $X'OX$ and $Y'OY$ to represent x -axis, y -axis respectively (see Fig. 22) and take 1 unit = 1 cm.
3. Fix a wire through O, in the vertical direction, representing the z -axis.
4. Fix nails of length 1 cm, 2 cm, 3 cm, 4 cm, etc. at different points on the squared paper (say at L (-2, -3), N (-2, 2), M (4, 1), S (3, -5)), etc.

Now the upper tips of these nails represent the points (say A, B, C, D) in the space.

DEMONSTRATION

1. Coordinates of the point A = (-2, -3, 1).
2. Coordinates of the point B = (-2, 2, 2).
3. Similarly find the coordinates of the point C and D.
4. By actual measurement (using a scale) distance AB = 5.1 cm.
5. By distance formula, $AB = \sqrt{(-2+2)^2 + (-3-2)^2 + (1-2)^2} = \sqrt{26} = 5.099$.

Thus, the distance AB, obtained by actual measurement is approximately same as the distance obtained by using the distance formula.

Same can be verified for other pairs of points A, C; B, C; A, D; C, D; B, D.

OBSERVATION

Coordinates of the point C = _____.

Coordinates of the point D = _____.

On actual measurement :

AC = _____, BC = _____.

AD = _____, CD = _____, BD = _____.

Using distance formula; $AC = \underline{\hspace{2cm}}$, $BC = \underline{\hspace{2cm}}$, $AD = \underline{\hspace{2cm}}$
 $CD = \underline{\hspace{2cm}}$, $BD = \underline{\hspace{2cm}}$.

Thus, the distance between two points in space obtained on actual measurement and by using distance formula is approximately the same.

APPLICATION

1. This activity is useful in visualising the position of different points in space (coordinates of points).
2. The concept of position vectors can also be explained through this activity.

Activity 23

OBJECTIVE

To demonstrate the equation of a plane in normal form.

MATERIAL REQUIRED

Two pieces of plywood of size $10 \text{ cm} \times 12 \text{ cm}$, a thin wooden rod with nuts and bolts fixed on both sides, 3 pieces of wires, pen/pencil.

METHOD OF CONSTRUCTION

1. Fix the wooden rod in between two wooden pieces with nuts and bolts so that the rod is perpendicular to the two wooden pieces. So, it represents the normal to the plane.
2. Take three wires and fix, them as shown in Fig. 23 so that \overline{OP} represents the vector \vec{a} and \overline{OA} represents \vec{r} . Then the wire \overline{PA} represents the vector $\vec{r} - \vec{a}$.

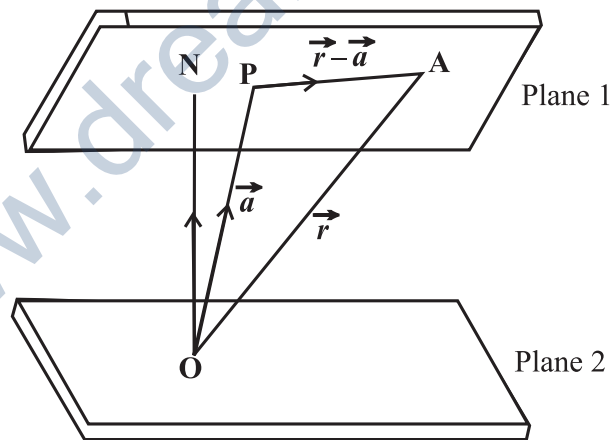


Fig. 23

DEMONSTRATION

1. The wire PA, i.e., the vector $(\vec{r} - \vec{a})$ lies on plane 1. On representing \vec{n} as normal to plane 1, \vec{n} is perpendicular to $(\vec{r} - \vec{a})$, normal to the plane.
2. Hence $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ which gives the equation of plane in the normal form.

OBSERVATION

1. \vec{a} is the position vector of _____, \vec{r} is the position vector of _____ vector \vec{n} is perpendicular to the vector _____.
2. $(\vec{r} - \vec{a}) \cdot \hat{n} = 0$, is the equation of the plane _____, in _____ form.

APPLICATION

This activity can also be utilised to show the position vector of a point in space (i.e., \vec{a} as position vector of O, \vec{r} the position vector of A).

Activity 24

OBJECTIVE

To verify that the angle between two planes is the same as the angle between their normals.

MATERIAL REQUIRED

Plywood pieces, wires, hinges.

METHOD OF CONSTRUCTION

1. Take two pieces of plywood 10 cm × 20 cm and join them with the help of hinges.
2. Fix two vertical wires on each plane to show normals to the planes.
3. Cut slots in the two planes to fix a third plywood piece showing third plane.

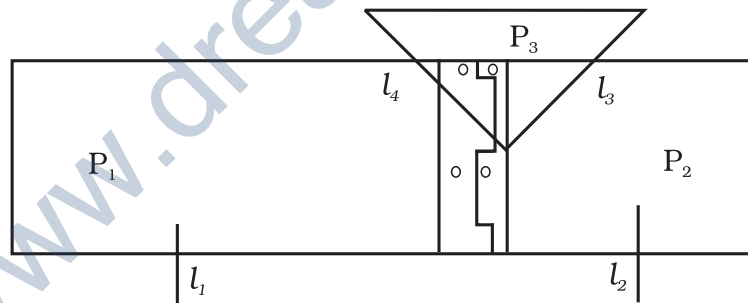


Fig. 24

DEMONSTRATION

1. P_1 represents the first plane.
2. P_2 represents the second plane.
3. Vertical wires l_1 and l_2 represent normals to the planes P_1 , P_2 , respectively.

4. l_3 and l_4 are the lines of intersections of the planes P_3 , with P_1 and P_2 , respectively.
5. Angle between lines l_3 and l_4 is the angle between the planes. It is same as the angle between their normals.

OBSERVATION

1. P_1 represents the _____.
2. P_2 represents the _____.
3. l_1 represents the _____.
4. l_2 represents the _____.
5. l_3 is the line of intersection _____.
6. l_4 is the line of intersection _____.
7. Angle between l_1 and l_2 is equal to _____.

APPLICATION

This model can also be used to find the angle between a line and a plane.

Activity 25

OBJECTIVE

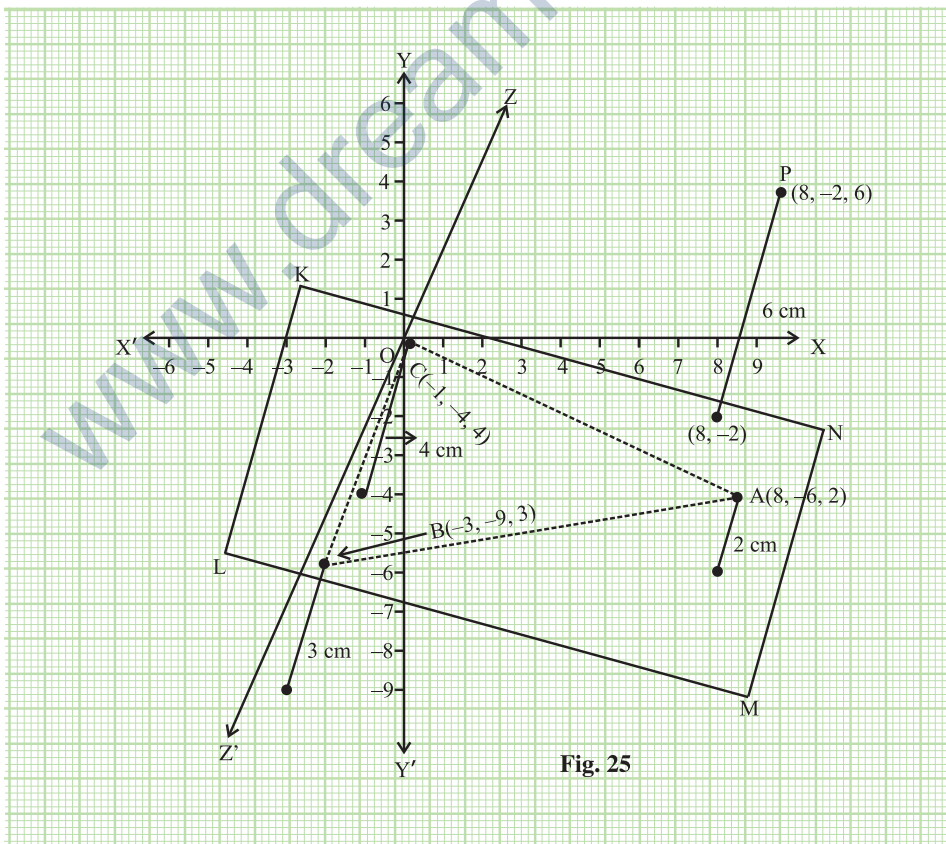
To find the distance of given point (in space) from a plane (passing through three non-collinear points) by actual measurement and also analytically.

MATERIAL REQUIRED

One cardboard of size 20 cm × 30 cm and another of size 10 cm × 15 cm., a thick sheet of paper of size 20 cm × 30 cm, nails of varying lengths with caps on one end, geometry box, wires.

METHOD OF CONSTRUCTION

1. Draw two mutually perpendicular lines $X'OX$ and $Y'OY$ on a thick sheet of paper representing x -axis, and y -axis, respectively intersecting at O , and graduate them.



- Paste this sheet on the cardboard of size 20 cm × 30 cm. Through O, fix a wire vertically to represent z -axis (see Fig. 25).
- Fix three nails of heights (say 2 cm, 3 cm and 4 cm) at three different points on this board (say at (8, -6), (-3, -9) and (-1, -4), respectively) (Fig. 25).
- The tips of these nails represent three points A, B and C in space.
- Now rest a plane KLMN represented by another cardboard on the tips of these three nails so that the points A, B, C, lie on the plane.
- Now fix a nail of length 6 cm at some point [say (8, -2)] on the cardboard. The tip of the nail will represent point P, from where the distance to the plane KLMN is to be found.

DEMONSTRATION

- Coordinates of the points A, B and C are (8, -6, 2), (-3, -9, 3), (-1, -4, 4), respectively.
- Coordinates of point P are (8, -2, 6).
- A set square is placed so that its one side-forming the right angle on the plane KLMN and the other side in the direction normal to the plane.
- Place a metre scale along the side of the set square which is in the direction normal to the plane KLMN and slide both of them until the metre scale touches the point P.
- The distance between the point P and the plane in the normal direction is measured using a metre scale.
- Equation of the plane passing through the points A, B, C is

$$\begin{vmatrix} x-8 & y+6 & z-2 \\ -3-8 & -9+6 & 3-2 \\ -1-8 & -4+6 & 4-2 \end{vmatrix} = 0 \text{ which is of the form } ax + by + cz + d = 0.$$

7. This distance is also calculated by using the formula

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

8. The two distance so obtained are the same.

OBSERVATION

1. The coordinates of A $(x_1, y_1, z_1) =$ _____.

B $(x_2, y_2, z_2) =$ _____.

C $(x_3, y_3, z_3) =$ _____.

Coordinates of point P = _____.

The distance(d) of P from the plane KLMN by actual measurement = _____.

2. Equation of plane through A, B, C using

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad \text{is} \quad \text{_____}.$$

The distance of P from this plane (represented by above equation)

using $d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} =$ _____.

Thus distance of a point P from a plane by actual measurement

= distance of P through analytical method = _____.

APPLICATION

1. With this activity it can be explained that through
 - (a) one point or through two points, infinite number of planes can pass,
 - (b) three non-collinear points, a unique plane passes.
2. This activity can also be used in explaining the concept of distance between two parallel planes.

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Activity 26

OBJECTIVE

To measure the shortest distance between two skew lines and verify it analytically.

MATERIAL REQUIRED

A piece of plywood of size $30\text{ cm} \times 20\text{ cm}$, a squared paper, three wooden blocks of size $2\text{ cm} \times 2\text{ cm} \times 2\text{ cm}$ each and one wooden block of size $2\text{ cm} \times 2\text{ cm} \times 4\text{ cm}$, wires of different lengths, set squares, adhesive, pen/pencil, etc.

METHOD OF CONSTRUCTION

1. Paste a squared paper on a piece of plywood.
2. On the squared paper, draw two lines OA and OB to represent x -axis, and y -axis, respectively.
3. Name the three blocks of size $2\text{ cm} \times 2\text{ cm} \times 2\text{ cm}$ as I, II and III. Name the other wooden block of size $2\text{ cm} \times 2\text{ cm} \times 4\text{ cm}$ as IV.
4. Place blocks I, II, III such that their base centres are at the points $(2, 2)$, $(1, 6)$ and $(7, 6)$, respectively, and block IV with its base centre at $(6, 2)$. Other wooden block of size $2\text{ cm} \times 2\text{ cm} \times 4\text{ cm}$ as IV.
5. Place a wire joining the points P and Q, the centres of the bases of the blocks I and III and another wire joining the centres R and S of the tops of blocks II and IV as shown in Fig. 26.
6. These two wires represent two skew lines.
7. Take a wire and join it perpendicularly with the skew lines and measure the actual distance.

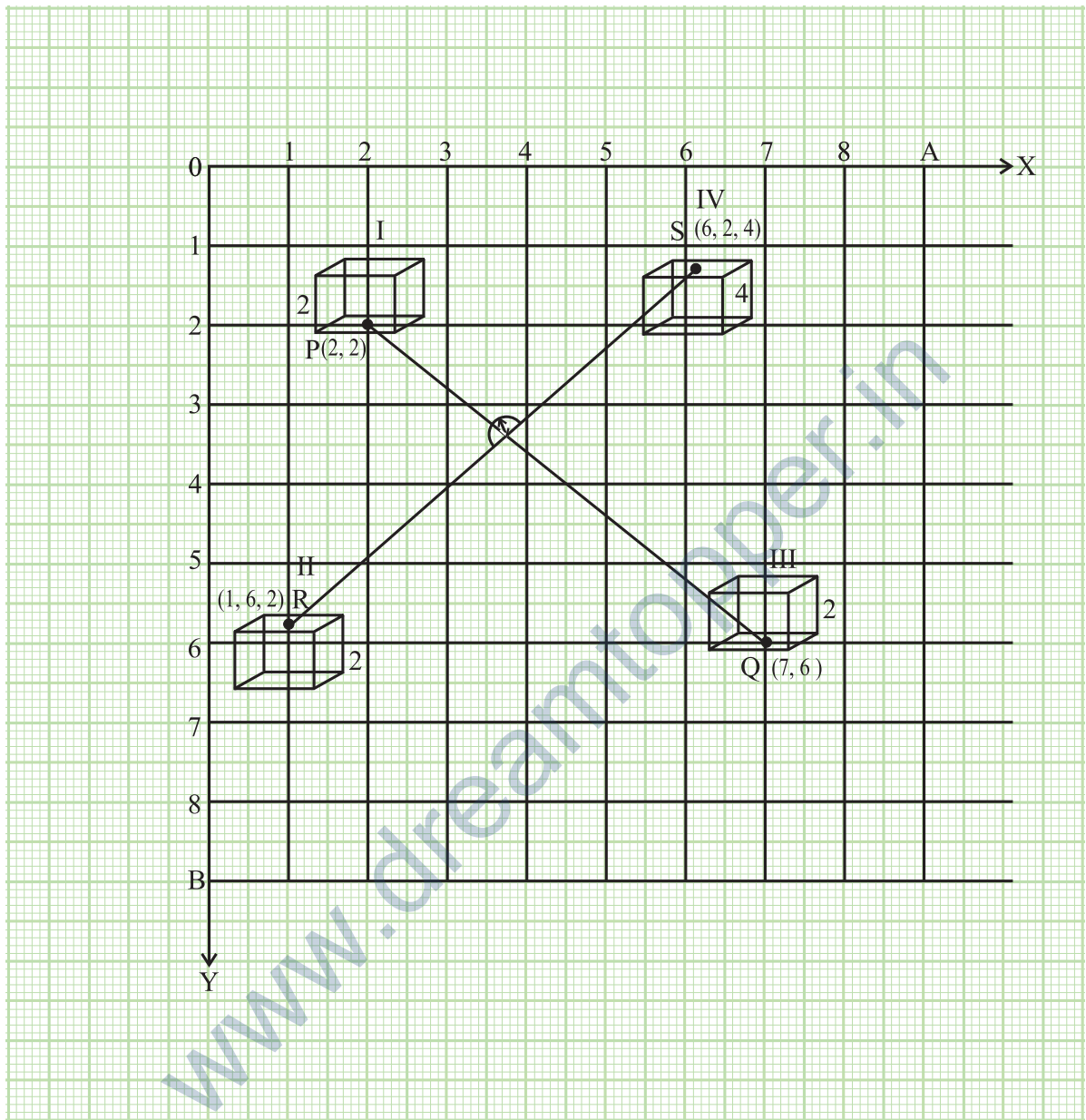


Fig. 26

DEMONSTRATION

1. A set-square is placed in such a way that its one perpendicular side is along the wire PQ.
2. Move the set-square along PQ till its other perpendicular side touches the other wire.

3. Measure the distance between the two lines in this position using set-square. This is the shortest distance between two skew lines.
4. Analytically, find the equation of line joining P (2, 2, 0) and Q (7, 6, 0) and other line joining R (1, 6, 2) and S (6, 2, 4) and find S.D. using $\frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$. The distance obtained in two cases will be the same.

OBSERVATION

1. Coordinates of point P are _____.
2. Coordinates of point Q are _____.
3. Coordinates of point R are _____.
4. Coordinates of point S are _____.
5. Equation of line PQ is _____.
6. Equation of line RS is _____.

Shortest distance between PQ and RS analytically = _____.

Shortest distance by actual measurement = _____.

The results so obtained are _____.

APPLICATION

This activity can be used to explain the concept of skew lines and of shortest distance between two lines in space.

Activity 27

OBJECTIVE

To explain the computation of conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice.

MATERIAL REQUIRED

A piece of plywood, white paper pen/pencil, scale, a pair of dice.

METHOD OF CONSTRUCTION

1. Paste a white paper on a piece of plywood of a convenient size.
2. Make a square and divide it into 36 unit squares of size 1cm each (see Fig. 27).
3. Write pair of numbers as shown in the figure.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Fig. 27

DEMONSTRATION

1. Fig. 27 gives all possible outcomes of the given experiment. Hence, it represents the sample space of the experiment.
2. Suppose we have to find the conditional probability of an event A if an event B has already occurred, where A is the event "a number 4 appears on both the dice" and B is the event "4 has appeared on at least one of the dice" i.e, we have to find $P(A | B)$.
3. From Fig. 27 number of outcomes favourable to A = 1
Number of outcomes favourable to B = 11
Number of outcomes favourable to $A \cap B = 1$.

4. (i) $P(B) = \frac{11}{36}$,

(ii) $P(A \cap B) = \frac{1}{36}$

(iii) $P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{11}$.

NOTE

1. You may repeat this activity by taking more events such as the probability of getting a sum 10 when a doublet has already occurred.

2. Conditional probability $P(A | B)$ can also be found by first taking the sample space of event B out of the sample space of the experiment, and then finding the probability A from it.

OBSERVATION

1. Outcome(s) favourable to A : _____, $n(A) =$ _____.
2. Outcomes favourable to B : _____, $n(B) =$ _____.
3. Outcomes favourable to $A \cap B$: _____, $n(A \cap B) =$ _____.
4. $P(A \cap B) =$ _____.
5. $P(A | B) =$ _____ = _____.

APPLICATION

This activity is helpful in understanding the concept of conditional probability, which is further used in Bayes' theorem.