The Purpose of the Mathematics Laboratory

National Policy on Education (1986) states "Mathematics should be visualised as a vehicle to train a child to think, analyse and articulate logically". National Curriculum Framework - 2005 brought out by NCERT states that the main goal of Mathematics education is mathematisation of child's thought process. These objectives can only be achieved if there is an opportunity of creating a scope of exploring, verifying and experimenting upon mathematical results by students themselves. Thus, there is need of adopting activity — oriented process rather than merely concentrating upon mastery of rules and formulae so as to do mathematical problems mechanically and pass out the examinations. There is need to provide the learners the scope for interaction, communication and representations of mathematical ideas by practising processes.

No doubt a laboratory is a place where scientific research and experiments are conducted for verification, exploration or discovery. Specifically, in mathematics the role of laboratory is helpful in understanding the mathematical concepts, formulae through activities. It is worth mentioning that pattern is central theme in mathematics which we need to develop practically to get insight into the mathematical concepts/theorems/formulae. Mathematics laboratory should not be solely a store house of teaching aids but in turn emphasis has to be laid on organising activities by students/teachers to rediscover the truth underlying the mathematical concepts. However, there may be a few interesting readymade geometrical and other models to motivate students. Moreover these models should be manipulative and dynamic.

A mathematics laboratory can foster mathematical awareness, skill building, positive attitude and learning by doing experiments in various topics of mathematics such as Algebra, Geometry, Mensuration, Trigonometry, Calculus, Coordinate Geometry, etc. It is the place where students can learn certain concepts using concrete objects and verify many mathematical facts and properties using models, measurements and other activities. It will also provide an opportunity to the students to do certain calculations using tables, calculators, etc., and also to listen or view certain audio-video cassettes relating to, remedial instructions, enrichment materials, etc. Mathematics laboratory will also provide an opportunity for the teacher to explain and demonstrate many mathematical concepts, facts and properties using concrete materials, models, charts, etc.

The teacher may also encourage students to prepare similar models and charts using materials like thermocol, cardboard, etc. in the laboratory. The laboratory will act as a forum for the teachers to discuss and deliberate on some important mathematical issues and problems of the day. It may also act as a place for teachers and the students to perform a number of mathematical celebrations and recreational activities.

Mathematics laboratory is expected to offer the following opportunities to learners:

- To discover the pattern for getting insight into the formulae
- To visualise algebraic and analytical results geometrically.
- To design practical demonstrations of mathematical results/formulae or the concepts.
- To encourage interactions amongst students and teachers through debate and discussions.
- To encourage students in recognising, extending, formulating patterns and enabling them to pose problems in the form of conjectures.
- To facilitate students in comprehending basic nature of mathematics from concrete to abstract.
- To provide opportunities to students of different ability groups in developing their skills of explaining and logical reasoning.
- To help students in constructing knowledge by themselves.
- To perform certain recreational activities in mathematics.
- To do certain projects under the proper guidance of the teacher.
- To explain visually some abstract concepts by using three dimensional models.
- To exhibit relatedness of mathematics with day to day life problems.

Role of Mathematics Laboratory in Teaching-Learning

Mathematics at Senior Secondary stage is a little more abstract as compared to the subject at the secondary stage. The mathematics laboratory at this stage can contribute in a big way to the learning of this subject.

Some of the ways are:

- Here the student will get an opportunity to understand the abstract ideas/ concepts through concrete objects and situations.
- The concepts of relations and functions can be easily understood by making working models and by making arrow diagrams using wires.
- Three dimensional concepts can only be conceived by three dimensional models in the laboratory, where as it is very difficult to understand these concepts on a black board.
- The concept of function and its inverse function, becomes very clear by drawing their graphs using mathematical instruments and using the concept of image about the line $y = x$, which can be done only in the laboratory.
- It provides greater scope for individual participation in the processes of learning and becoming autonomous learner.
- In the laboratory a student is encouraged to think, discuss with others and with the teacher. Thus, he can assimilate the concepts in a more effective manner.
- To the teacher also, mathematics laboratory enables to demonstrate and explain the abstract mathematical ideas, in a better way by using concrete objects, models etc.

Management and Maintenance of Laboratory

There is no second opinion that for effective teaching and learning 'Learning by doing' is of great importance as the experiences gained remains permanently affixed in the mind of the child. Exploring what mathematics is about and arriving at truth provides for pleasure of doing, understanding, developing positive attitude, and learning processes of mathematics and above all the great feeling of attachment with the teacher as facilitator. It is said 'a bad teacher teaches the truth but a good teacher teaches how to arrive at the truth.

A principle or a concept learnt as a conclusion through activities under the guidance of the teacher stands above all other methods of learning and the theory built upon it, can not be forgotten. On the contrary, a concept stated in the classroom and verified later on in the laboratory doesn't provide for any great experience nor make child's curiosity to know any good nor provides for any sense of achievement.

A laboratory is equipped with instruments, apparatus, equipments, models apart from facilities like water, electricity, etc. Non availability of a single material or facility out of these may hinder the performance of any experiment activity in the laboratory. Therefore, the laboratory must be well managed and well maintained.

A laboratory is managed and maintained by persons and the material required. Therefore, management and maintenance of a laboratory may be categorised as the personal management and maintenance and the material management and maintenance.

(A) Personal Management and Maintenance

The persons who manage and maintain laboratories are generally called laboratory assistant and laboratory attendant. Collectively they are known as laboratory staff. Teaching staff also helps in managing and maintenance of the laboratory whenever and wherever it is required.

In personal management and maintenance following points are considered:

1. **Cleanliness**

   A laboratory should always be neat and clean. When students perform experiment activities during the day, it certainly becomes dirty and things are scattered. So, it is the duty of the lab staff to clean the laboratory when the day's work is over and also place the things at their proper places if these are lying scattered.

2. **Checking and arranging materials for the day's work**

   Lab staff should know that what activities are going to be performed on a particular day. The material required for the day's activities must be arranged one day before.

   The materials and instruments should be arranged on tables before the class comes to perform an activity or the teacher brings the class for a demonstration.

3. The facilities like water, electricity, etc. must be checked and made available at the time of experiments.
4. It is better if a list of materials and equipments is pasted on the wall of the laboratory.
5. Many safety measures are required while working in laboratory. A list of such measures may be pasted on a wall of the laboratory.
6. While selecting the laboratory staff, the school authority must see that the persons should have their education with mathematics background.
7. A days training of 7 to 10 days may be arranged for the newly selected laboratory staff with the help of mathematics teachers of the school or some resource persons outside the school.
8. A first aid kit may be kept in the laboratory.

(B) Management and Maintenance of Materials

A laboratory requires a variety of materials to run it properly. The quantity of materials however depends upon the number of students in the school.

To manage and maintain materials for a laboratory following points must be considered:

1. A list of instruments, apparatus, activities and material may be prepared according to the experiments included in the syllabus of mathematics.
2. A group of mathematics teachers may visit the agencies or shops to check the quality of the materials and compare the rates. This will help to acquire the material of good quality at appropriate rates.

3. The materials required for the laboratory must be checked from time to time. If some materials or other consumable things are exhausted, orders may be placed for the same.
4. The instruments, equipments and apparatus should also be checked regularly by the laboratory staff. If any repair is required it should be done immediately. If any part is to be replaced, it should be ordered and replaced.
5. All the instruments, equipments, apparatus, etc. must be stored in the almirahs and cupboards in the laboratory or in a separate store room.

Equipment for Mathematics Laboratory at the Higher Secondary Stage

As the students will be involved in a lot of model making activities under the guidance of the teacher, the smooth running of the mathematics laboratory will depend upon the supply of oddments such as strings and threads, cellotape, white cardboard, hardboard, needles and pins, drawing pins, sandpaper, pliers, screw-drivers, rubber bands of different colours, gummed papers and labels, squared papers, plywood, scissors, saw, paint, soldering, solder wire, steel wire, cotton wool, tin and plastic sheets, glazed papers, etc. Besides these, some models, charts, slides, etc., made up of a good durable material should also be there for the teacher to demonstrate some mathematical concepts, facts and properties before the students. Different tables, ready reckner should also be there (in the laminated form) so that these can be used by the students for different purposes. Further, for performing activities such as measuring, drawing and calculating, consulting reference books, etc., there should be equipments like mathematical instruments, calculators, computers, books, journals mathematical dictionaries etc., in the laboratory.

In view of the above, following is the list of suggested instruments/models for the laboratory:

EQUIPMENT

Mathematical instrument set (Wooden Geometry Box for demonstration containing rulers, set-squares, divider, protractor and compasses), some geometry boxes, metre scales of 100 cm, 50 cm and 30 cm, measuring tape, diagonal scale, clinometer, calculators, computers including related software etc.

MODELS FOR DEMONSTRATION OF–

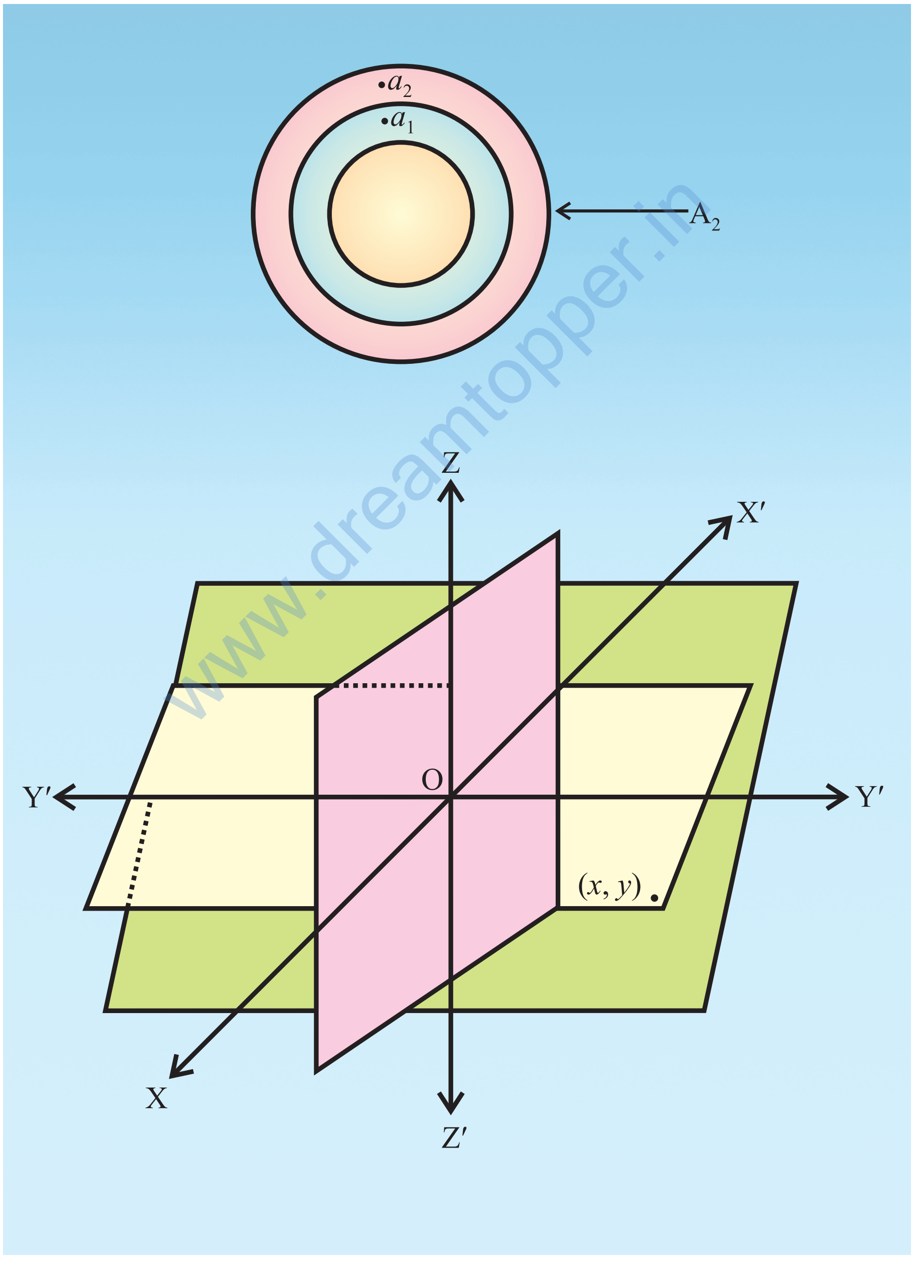
- Sets
- Relations and Functions
- Quadratic functions with the help of linear functions
- Sequence and series
- Pascal's triangle
- Arithmetic Progression

- Conic Sections
- Increasing, decreasing functions
- Maxima, minima, point of inflection
- Lagrange's minima, point of inflection
- Rolle's theorem
- Definite Integral as limit of sum
- Angle in semicircle using vectors
- Construction of parabola when distance between directrix and focus is given
- Construction of ellipse when major and minor axes are given
- Octants
- Shortest distance between two skew lines
- Geometrical interpretation of scalar and vector product
- Equation of a straight line passing through a fixed point and parallel to a given vector
- Equation to a plane
- Angle between two planes
- Bisection of the angles between two planes by a third plane
- Intersection of three planes
- Projection of the line segment
- Sample spaces
- Conditional Probability

STATIONERY AND ODDMENTS

Rubber-bands of different colours, Marbles of different colours, a pack of playing cards, graph paper/ squared paper, dotted paper, drawing pins, erasers, pencils, sketch pens, cellotapes, threads of different colours, glazed papers, kite papers, tracing papers, adhesive, pins, scissors and cutters, hammers, saw, thermocol sheets, sand paper, nails and screws of different sizes, screw drivers, drill machine with bit set, and pliers.

Activities for
Class XI



Mathematics is one of the most important cultural components of every modern society. Its influence another cultural element has been so fundamental and wide-spread as to warrant the statement that her "most modern" ways of life would hardly have been possilbly without mathematics. Appeal to such obvious examples as electronics radio, television, computing machines, and space travel, to substantiate this statement is unnecessary : the elementary art of calculating is evidence enough. Imagine trying to get through three day without using numbers in some fashion or other!

– R.L. Wilder

Activity 1

OBJECTIVE

To find the number of subsets of a given set and verify that if a set has n number of elements, then the total number of subsets is 2^n.

MATERIAL REQUIRED

Paper, different coloured pencils.

METHOD OF CONSTRUCTION

1. Take the empty set (say) A_0 which has no element.
2. Take a set (say) A_1 which has one element (say) a_1.
3. Take a set (say) A_2 which has two elements (say) a_1 and a_2.
4. Take a set (say) A_3 which has three elements (say) a_1, a_2 and a_3.

DEMONSTRATION

1. Represent A_0 as in Fig. 1.1

   Here the possible subsets of A_0 is A_0 itself only, represented symbolically by ϕ. The number of subsets of A_0 is $1 = 2^0$.

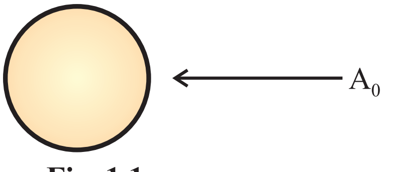


Fig. 1.1

2. Represent A_1 as in Fig. 1.2. Here the subsets of A_1 are ϕ, $\{a_1\}$. The number of subsets of A_1 is $2 = 2^1$

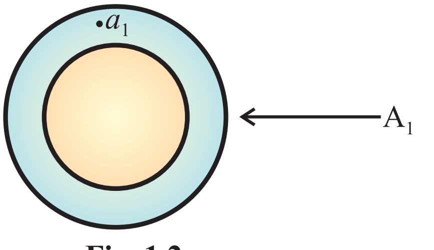


Fig. 1.2

3. Represent A_2 as in Fig. 1.3

   Here the subsets of A_2 are ϕ, $\{a_1\}$, $\{a_2\}$, $\{a_1, a_2\}$. The number of subsets of A_2 is $4 = 2^2$.

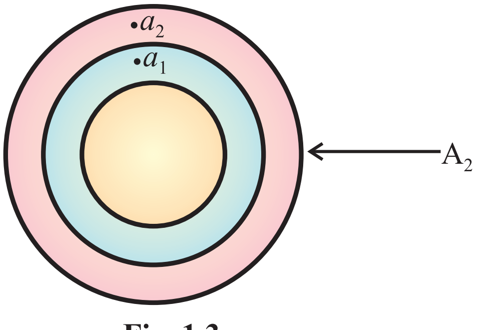


Fig. 1.3

4. Represent A_3 as in Fig. 1.4

   Here the subsets of A_3 are ϕ, $\{a_1\}$, $\{a_2\}$, $\{a_3)$,$\{a_1, a_2\}$, $\{a_2, a_3)$,$\{a_3, a_1)$ and $\{a_1, a_2, a_3\}$. The number of subsets of A_3 is $8 = 2^3$.

5. Continuing this way, the number of subsets of set A containing n elements $a_1, a_2, ..., a_n$ is 2^n.

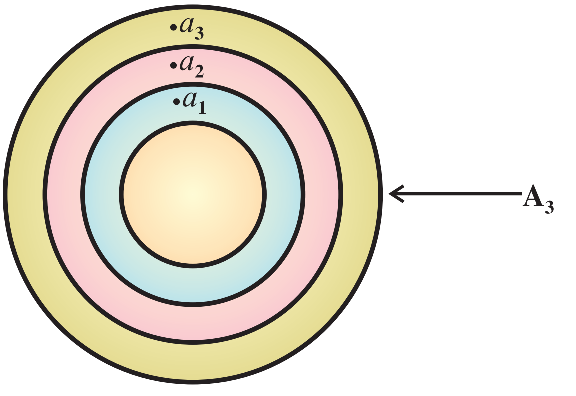


Fig. 1.4

OBSERVATION

1. The number of subsets of A_0 is ___________ $= 2^{\square}$
2. The number of subsets of A_1 is ___________ $= 2^{\square}$
3. The number of subsets of A_2 is ___________ $= 2^{\square}$
4. The number of subsets of A_3 is ___________ $= 2^{\square}$
5. The number of subsets of A_{10} is $= 2^{\square}$
6. The number of subsets of A_n is $= 2^{\square}$

APPLICATION

The activity can be used for calculating the number of subsets of a given set.

Activity 2

Objective

To verify that for two sets A and B, n (A×B) = pq and the total number of relations from A to B is 2^{pq}, where $n(\text{A}) = p$ and $n(\text{B}) = q$.

Material Required

Paper, different coloured pencils.

Method of Construction

1. Take a set A_1 which has one element (say) a_1, and take another set B_1, which has one element (say) b_1.
2. Take a set A_2 which has two elements (say) a_1 and a_2 and take another set B_3, which has three elements (say) b_1, b_2 and b_3.
3. Take a set A_3 which has three elements (say) a_1, a_2 and a_3, and take another set B_4, which has four elements (say) b_1, b_2, b_3 and b_4.

Demonstration

1. Represent all the possible correspondences of the elements of set A_1 to the elements of set B_1 visually as shown in Fig. 2.1.

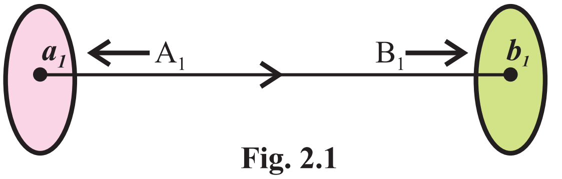


Fig. 2.1

2. Represent all the possible correspondences of the elements of set A_2 to the elements of set B_3 visually as shown in Fig. 2.2.

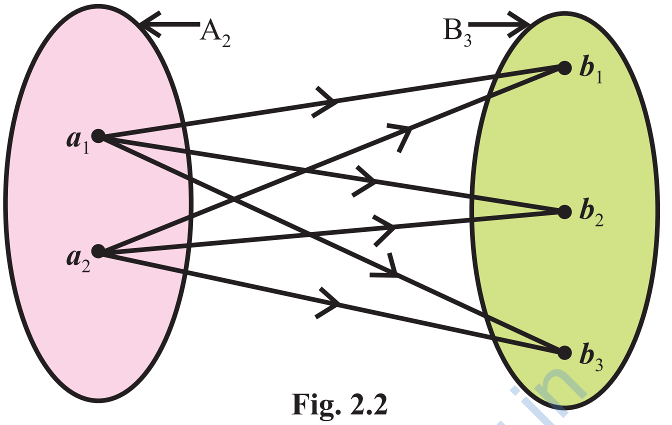


Fig. 2.2

3. Represent all the possible correspondences of the elements of set A_3 to the elements of set B_4 visually as shown in Fig. 2.3.

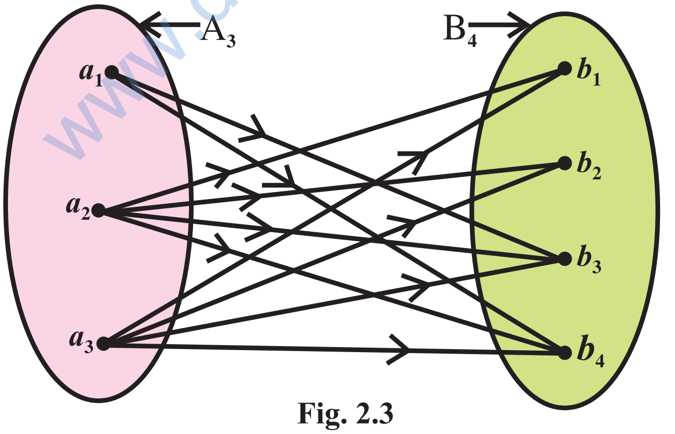


Fig. 2.3

4. Similar visual representations can be shown between the elements of any two given sets A and B.

OBSERVATION

1. The number of arrows, i.e., the number of elements in cartesian product $(A_1 \times B_1)$ of the sets A_1 and B_1 is _ × _ and the number of relations is $2^{\square}$.
2. The number of arrows, i.e., the number of elements in cartesian product $(A_2 \times B_3)$ of the sets A_2 and B_3 is _ × _ and number of relations is $2^{\square}$.
3. The number of arrows, i.e., the number of elements in cartesian product $(A_3 \times B_4)$ of the sets A_3 and B_4 is _ × _ and the number of relations is $2^{\square}$.

NOTE

The result can be verified by taking other sets A_4, A_5, ..., A_p, which have elements 4, 5,..., p, respectively, and the sets B_5, B_6, ..., B_q which have elements 5, 6,..., q, respectively. More precisely we arrive at the conclusion that in case of given set A containing p elements and the set B containing q elements, the total number of relations from A to B is 2^{pq}, where $n(A \times B) = n(A)\, n(B) = pq$.

Activity 3

OBJECTIVE

To represent set theoretic operations using Venn diagrams.

MATERIAL REQUIRED

Hardboard, white thick sheets of paper, pencils, colours, scissors, adhesive.

METHOD OF CONSTRUCTION

1. Cut rectangular strips from a sheet of paper and paste them on a hardboard. Write the symbol U in the left/right top corner of each rectangle.
2. Draw circles A and B inside each of the rectangular strips and shade/colour different portions as shown in Fig. 3.1 to Fig. 3.10.

DEMONSTRATION

1. U denotes the universal set represented by the rectangle.
2. Circles A and B represent the subsets of the universal set U as shown in the figures 3.1 to 3.10.
3. A' denote the complement of the set A, and B' denote the complement of the set B as shown in the Fig. 3.3 and Fig. 3.4.
4. Coloured portion in Fig. 3.1. represents $A \cup B$.

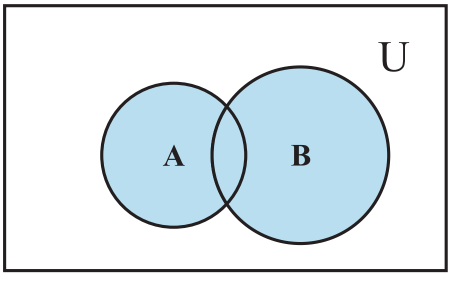


Fig. 3.1

5. Coloured portion in Fig. 3.2. represents A ∩ B.

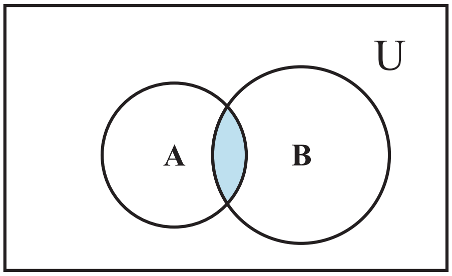


Fig. 3.2

6. Coloured portion in Fig. 3.3 represents A′

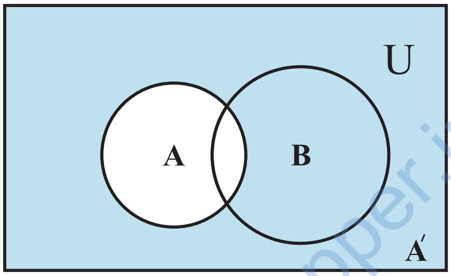


Fig. 3.3

7. Coloured portion in Fig. 3.4 represents B′

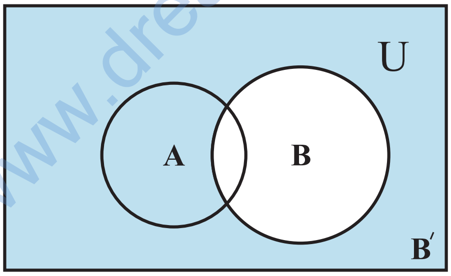


Fig. 3.4

8. Coloured portion in Fig. 3.5 represents (A ∩ B)′

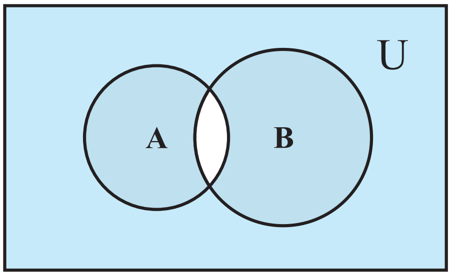


Fig. 3.5

9. Coloured portion in Fig. 3.6 represents $(A \cup B)'$

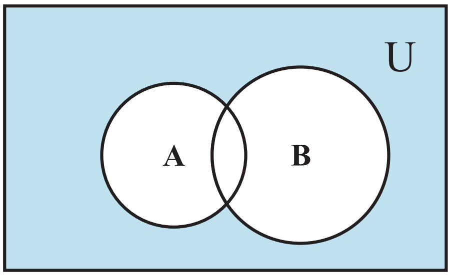


Fig. 3.6

10. Coloured portion in Fig. 3.7 represents $A' \cap B$ which is same as B – A.

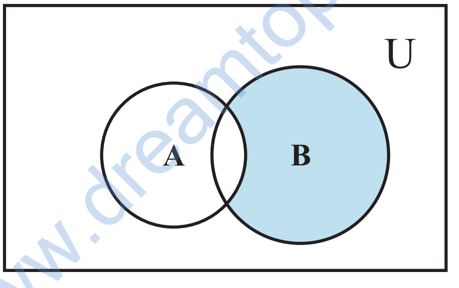


Fig. 3.7

11. Coloured portion in Fig. 3.8 represents $A' \cup B$.

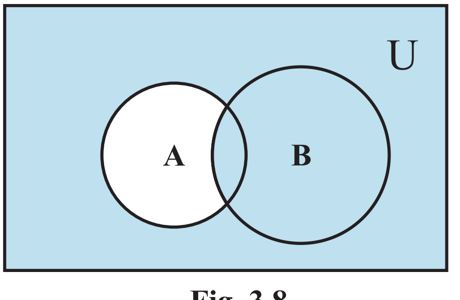


Fig. 3.8

12. Fig. 3.9 shows $A \cap B = \phi$

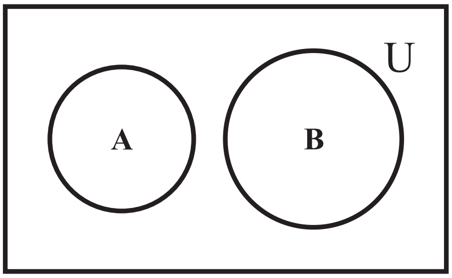


Fig. 3.9

13. Fig. 3.10 shows $A \subset B$

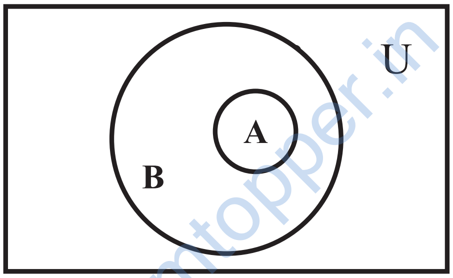


Fig. 3.10

Observation

1. Coloured portion in Fig. 3.1, represents _______________
2. Coloured portion in Fig. 3.2, represents _______________
3. Coloured portion in Fig. 3.3, represents _______________
4. Coloured portion in Fig. 3.4, represents _______________
5. Coloured portion in Fig. 3.5, represents _______________
6. Coloured portion in Fig. 3.6, represents _______________
7. Coloured portion in Fig. 3.7, represents _______________
8. Coloured portion in Fig. 3.8, represents _______________
9. Fig. 3.9, shows that $(A \cap B) =$ _______________
10. Fig. 3.10, represents A _______________ B.

Application

Set theoretic representation of Venn diagrams are used in Logic and Mathematics.

Activity 4

Objective

To verify distributive law for three given non-empty sets A, B and C, that is, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Material Required

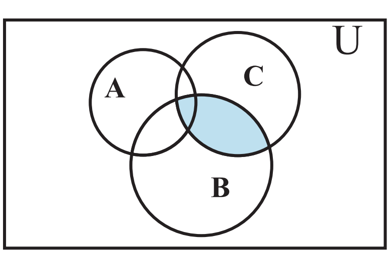
Hardboard, white thick sheets of paper, pencil, colours, scissors, adhesive.

Method of Construction

1. Cut five rectangular strips from a sheet of paper and paste them on the hardboard in such a way that three of the rectangles are in horizontal line and two of the remaining rectangles are also placed horizontally in a line just below the above three rectangles. Write the symbol U in the left/right top corner of each rectangle as shown in Fig. 4.1, Fig. 4.2, Fig. 4.3, Fig. 4.4 and Fig. 4.5.
2. Draw three circles and mark them as A, B and C in each of the five rectangles as shown in the figures.
3. Colour/shade the portions as shown in the figures.

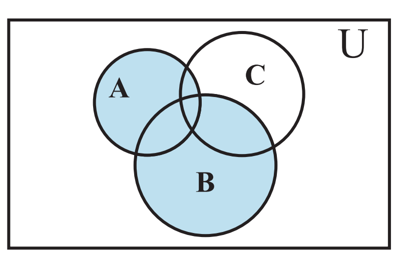
Demonstration

1. U denotes the universal set represented by the rectangle in each figure.
2. Circles A, B and C represent the subsets of the universal set U.



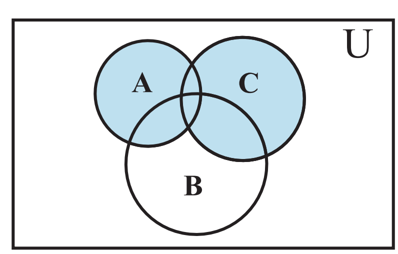
$B \cap C$

Fig. 4.1



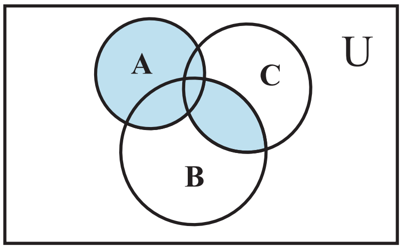
$A \cup B$

Fig. 4.2



$A \cup C$

Fig. 4.3



$A \cup (B \cap C)$

Fig. 4.4

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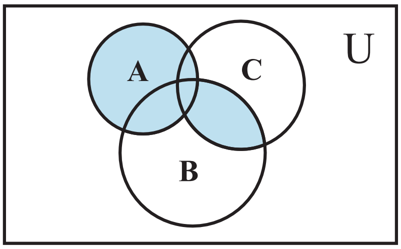

$(A \cup B) \cap (A \cup C)$

Fig. 4.5

3. In Fig. 4.1, coloured/shaded portion represents $B \cap C$, coloured portions in Fig. 4.2 represents $A \cup B$, Fig. 4.3 represents $A \cup C$, Fig. 4.4 represents $A \cup (B \cap C)$ and coloured portion in Fig. 4.5 represents $(A \cup B) \cap (A \cup C)$.

OBSERVATION

1. Coloured portion in Fig. 4.1 represents __________.
2. Coloured portion in Fig. 4.2, represents __________.
3. Coloured portion in Fig. 4.3, represents __________.
4. Coloured portion in Fig. 4.4, represents __________.
5. Coloured portion in Fig. 4.5, represents __________.
6. The common coloured portions in Fig. 4.4 and Fig. 4.5 are __________.
7. $A \cup (B \cap C) =$ __________.

Thus, the distributive law is verified.

APPLICATION

Distributivity property of set operations is used in the simplification of problems involving set operations.

NOTE

In the same way, the other distributive law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ can also be verified.

Activity 5

Objective

To identify a relation and a function.

Material Required

Hardboard, battery, electric bulbs of two different colours, testing screws, tester, electrical wires and switches.

Method of Construction

1. Take a piece of hardboard of suitable size and paste a white paper on it.
2. Drill eight holes on the left side of board in a column and mark them as A, B, C, D, E, F, G and H as shown in the Fig.5.
3. Drill seven holes on the right side of the board in a column and mark them as P, Q, R, S, T, U and V as shown in the Figure 5.
4. Fix bulbs of one colour in the holes A, B, C, D, E, F, G and H.
5. Fix bulbs of the other colour in the holes P, Q, R, S, T, U and V.

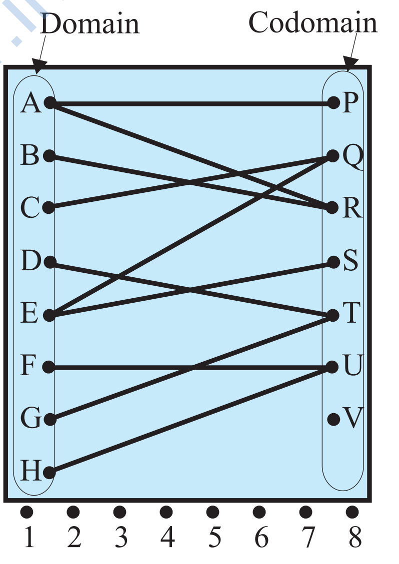

Fig. 5

6. Fix testing screws at the bottom of the board marked as 1, 2, 3, ..., 8.
7. Complete the electrical circuits in such a manner that a pair of corresponding bulbs, one from each column glow simultaneously.
8. These pairs of bulbs will give ordered pairs, which will constitute a relation which in turn may /may not be a function [see Fig. 5].

DEMONSTRATION

1. Bulbs at A, B, ..., H, along the left column represent domain and bulbs along the right column at P, Q, R, ..., V represent co-domain.
2. Using two or more testing screws out of given eight screws obtain different order pairs. In Fig.5, all the eight screws have been used to give different ordered pairs such as (A, P), (B, R), (C, Q) (A, R), (E, Q), etc.
3. By choosing different ordered pairs make different sets of ordered pairs.

OBSERVATION

1. In Fig.5, ordered pairs are _________.
2. These ordered pairs constitute a _______.
3. The ordered pairs (A, P), (B, R), (C, Q), (E, Q), (D, T), (G, T), (F, U), (H, U) constitute a relation which is also a _____.
4. The ordered pairs (B, R), (C, Q), (D, T), (E, S), (E, Q) constitute a _____ which is not a _____.

APPLICATION

The activity can be used to explain the concept of a relation or a function. It can also be used to explain the concept of one-one, onto functions.

Activity 6

OBJECTIVE

To distinguish between a Relation and a Function.

MATERIAL REQUIRED

Drawing board, coloured drawing sheets, scissors, adhesive, strings, nails etc.

METHOD OF CONSTRUCTION

1. Take a drawing board/a piece of plywood of convenient size and paste a coloured sheet on it.
2. Take a white drawing sheet and cut out a rectangular strip of size 6 cm × 4 cm and paste it on the left side of the drawing board (see Fig. 6.1).

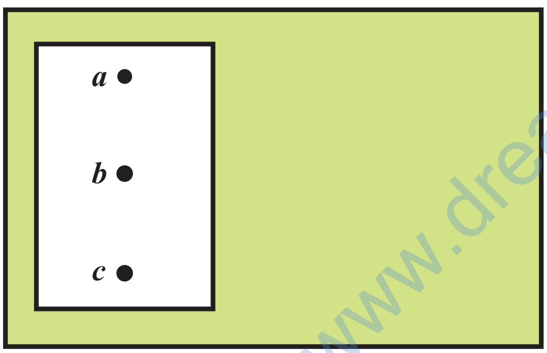

Fig. 6.1

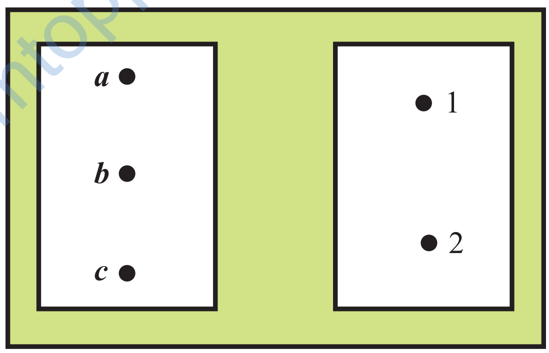

Fig. 6.2

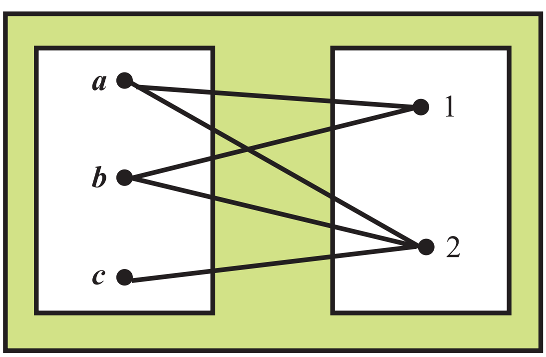

Fig. 6.3

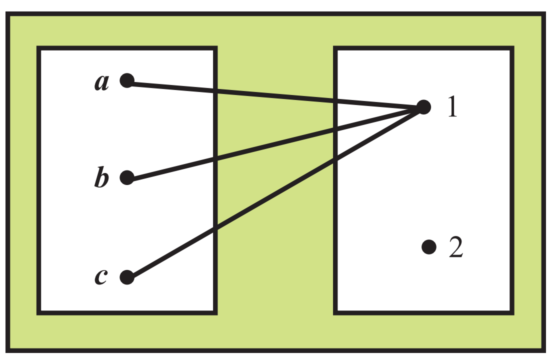

Fig. 6.4

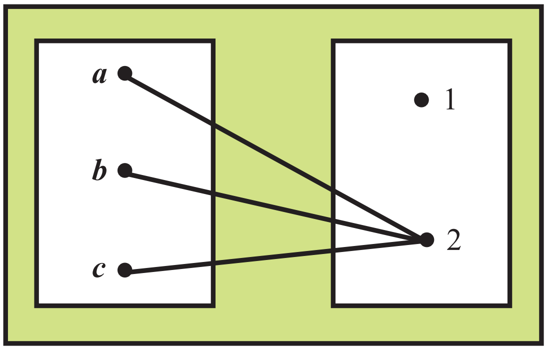

Fig. 6.5

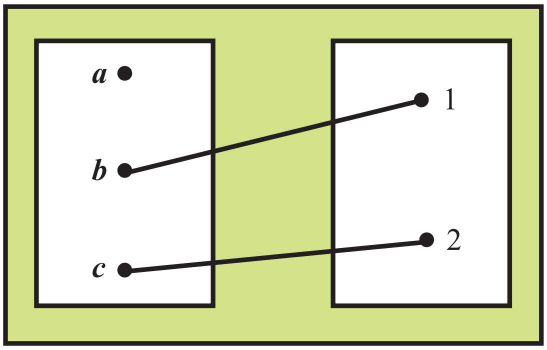

Fig. 6.6

3. Fix three nails on this strip and mark them as *a, b, c* (see Fig. 6.1).
4. Cut out another white rectangular strip of size 6 cm × 4 cm and paste it on the right hand side of the drawing board.
5. Fix two nails on the right side of this strip (see Fig. 6.2) and mark them as 1 and 2.

Demonstration

1. Join nails of the left hand strip to the nails on the right hand strip by strings in different ways. Some of such ways are shown in Fig. 6.3 to Fig. 6.6.
2. Joining nails in each figure constitute different ordered pairs representing elements of a relation.

Observation

1. In Fig. 6.3, ordered pairs are _____________.

   These ordered pairs constitute a ____________ but not a __________.
2. In Fig. 6.4, ordered pairs are ___________. These constitute a ________ as well as _________.
3. In Fig 6.5, ordered pairs are ________. These ordered pairs constitute a _________ as well as _________.
4. In Fig. 6.6, ordered pairs are _________. These ordered pairs do not represent _______ but represent _________.

APPLICATION

Such activity can also be used to demonstrate different types of functions such as constant function, identity function, injective and surjective functions by joining nails on the left hand strip to that of right hand strip in suitable manner.

NOTE

In the above activity nails have been joined in some different ways. The student may try to join them in other different ways to get more relations of different types. The number of nails can also be changed on both sides to represent different types of relations and functions.

Activity 7

OBJECTIVE

To verify the relation between the degree measure and the radian measure of an angle.

MATERIAL REQUIRED

Bangle, geometry box, protractor, thread, marker, cardboard, white paper.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw a circle using a bangle on the white paper.
3. Take a set square and place it in two different positions to find diameters PQ and RS of the circle as shown in the Fig.7.1 and 7.2

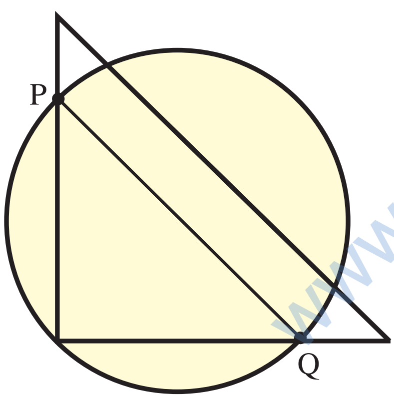

Fig. 7.1

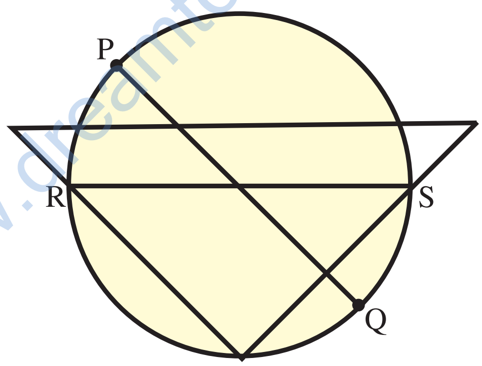

Fig. 7.2

4. Let PQ and RS intersect at C. The point C will be the centre of the circle (Fig. 7.3).
5. Clearly CP = CR = CS = CQ = radius.

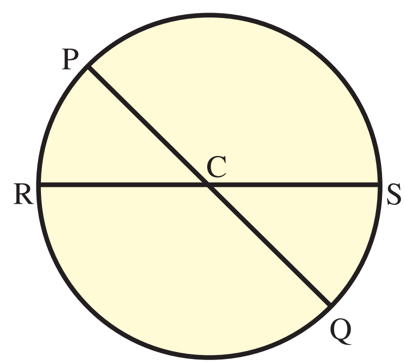

Fig. 7.3

DEMONSTRATION

1. Let the radius of the circle be r and l be an arc subtending an angle θ at the centre C, as shown in Fig. 7.4. $\theta = \frac{l}{r}$ radians.

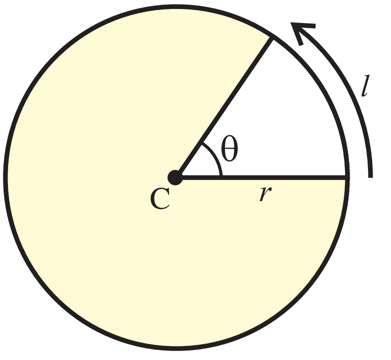

Fig. 7.4

2. If Degree measure of $\theta = \frac{l}{2\pi r} \times 360$ degrees

Then $\frac{l}{r}$ radians $= \frac{l}{2\pi r} \times 360$ degrees

or 1 radian $= \frac{180}{\pi}$ degrees = 57.27 degrees.

OBSERVATION

Using thread, measure arc lengths RP, PS, RQ, QS and record them in the table given below :

S.No	Arc	length of arc (l)	radius of circle (r)	Radian measure
1.	$\overset{\frown}{RP}$	--------	--------	$\angle RCP = \frac{\overset{\frown}{RP}}{r} =$ __
2.	$\overset{\frown}{PS}$	--------	--------	$\angle PCS = \frac{\overset{\frown}{PS}}{r} =$ __
3.	$\overset{\frown}{SQ}$	--------	--------	$\angle SCQ = \frac{\overset{\frown}{SQ}}{r} =$ __
4.	$\overset{\frown}{QR}$	--------	--------	$\angle QCR = \frac{\overset{\frown}{QR}}{r} =$ __

2. Using protractor, measure the angle in degrees and complete the table.

Angle	Degree measure	Radian Measure	Ratio $= \dfrac{\textit{Degree measure}}{\textit{Radian measure}}$
$\angle$ RCP	--------	--------	--------
$\angle$ PCS	--------	--------	--------
$\angle$ QCS	--------	--------	--------
$\angle$ QCR	--------	--------	--------

3. The value of one radian is equal to _________ degrees.

APPLICATION

This result is useful in the study of trigonometric functions.

Activity 8

Objective

To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant.

Material Required

Cardboard, white chart paper, ruler, coloured pens, adhesive, steel wires and needle.

Method of Construction

1. Take a cardboard of convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on chart paper.
3. Through the centre of the circle, draw two perpendicular lines $X'OX$ and YOY' representing x-axis and y-axis, respectively, as shown in Fig.8.1.

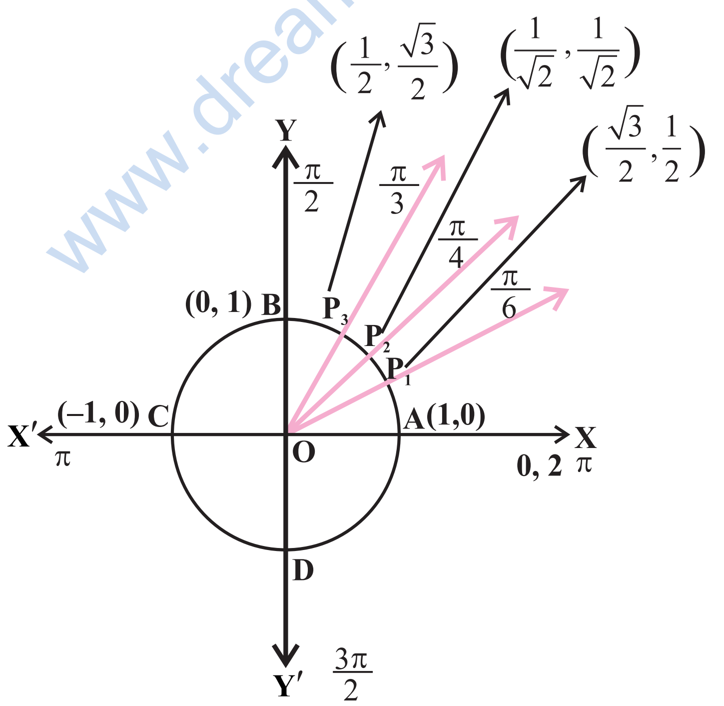

Fig. 8.1

4. Mark the points as A, B, C and D, where the circle cuts the x-axis and y-axis, respectively, as shown in Fig. 8.1.

5. Through O, draw angles P_1OX, P_2OX, and P_3OX of measures $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$, respectively.

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.

Demonstration

1. The coordinates of the point P_1 are $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ because its x-coordinate is $\cos\frac{\pi}{6}$ and y-coordinate is $\sin\frac{\pi}{6}$. The coordinates of the points P_2 and P_3 are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, respectively.

2. To find the value of sine or cosine of some angle in the second quadrant (say) $\frac{2\pi}{3}$, rotate the needle in anti clockwise direction making an angle P_4OX of measure $\frac{2\pi}{3} = 120°$ with the positive direction of x-axis.

3. Look at the position OP_4 of the needle in

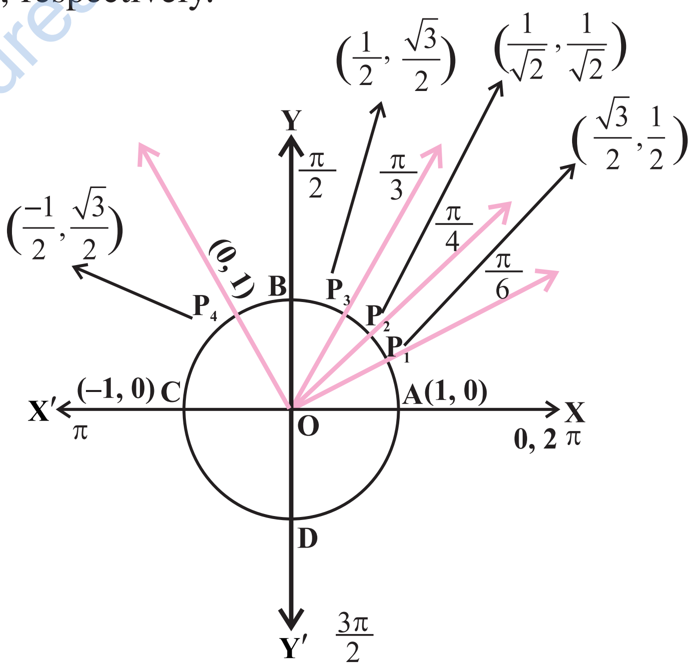

Fig. 8.2

Fig.8.2. Since $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, OP_4 is the mirror image of OP_3 with respect to y-axis. Therefore, the coordinate of P_4 are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Thus $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{2\pi}{3} = -\frac{1}{2}$.

4. To find the value of sine or cosine of some angle say, $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$, i.e., $\frac{-2\pi}{3}$ (say) in the third quadrant, rotate the needle in anti clockwise direction making as an angle of $\frac{4\pi}{3}$ with the positive direction of x-axis.

5. Look at the new position OP_5 of the needle, which is shown in Fig. 8.3. Point P_5 is the mirror image of the point P_4 (since $\angle P_4OX' = P_5OX'$) with respect to x-axis. Therefore, co-ordinates of P_5 are $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ and hence

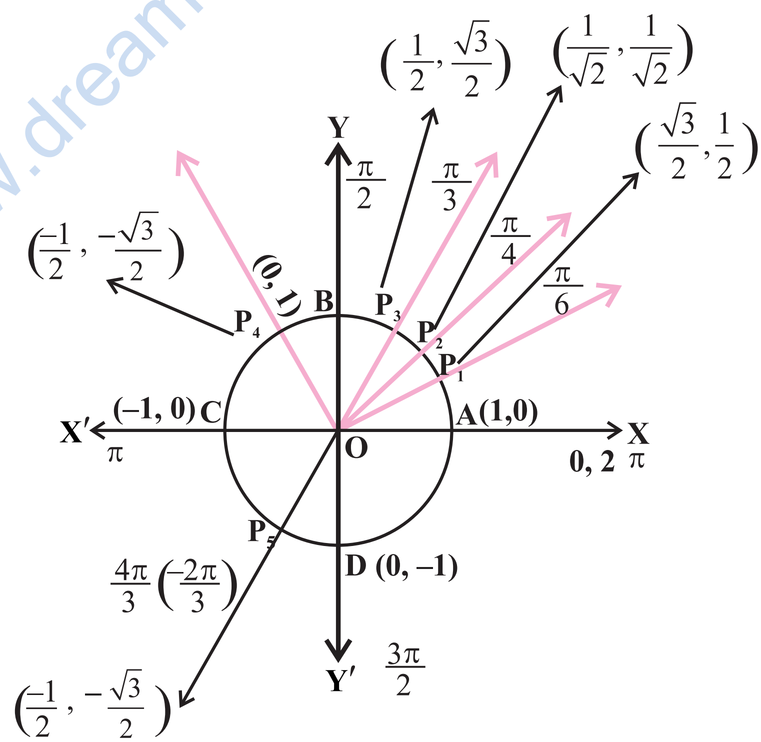

Fig. 8.3

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}.$$

6. To find the value of sine or cosine of some angle in the fourth quadrant, say $\frac{7\pi}{4}$, rotate the needle in anti clockwise direction making an angle of $\frac{7\pi}{4}$ with the positive direction of x-axis represented by OP_6, as shown in Fig. 8.4. Angle $\frac{7\pi}{4}$ in anti clockwise direction = Angle $-\frac{\pi}{4}$ in the clockwise direction.

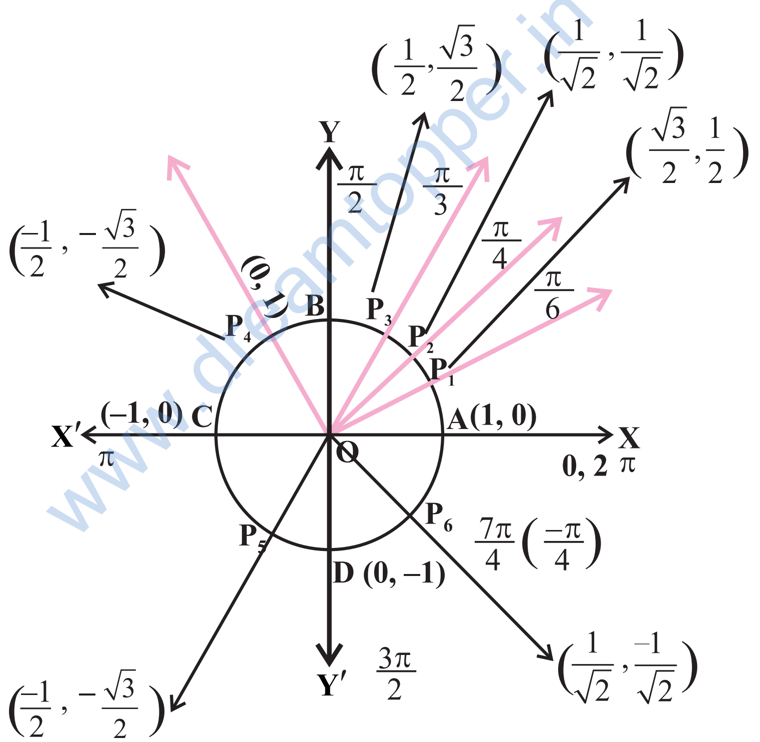

Fig. 8.4

From Fig. 8.4, P_6 is the mirror image of P_2 with respect to x-axis. Therefore, coordinates of P_6 are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Thus $\sin\left(\frac{7\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

and $\cos\left(\frac{7\pi}{4}\right)=\cos\left(-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

8. To find the value of sine or cosine of some angle, which is greater than one revolution, say $\frac{13\pi}{6}$, rotate the needle in anti clockwise direction since $\frac{13\pi}{6}=2\pi+\frac{\pi}{6}$, the needle will reach at the position OP_1. Therefore, $\sin\left(\frac{13\pi}{6}\right)=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $\cos\left(\frac{13\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

Observation

1. Angle made by the needle in one complete revolution is _________.
2. $\cos\frac{\pi}{6}=$ _________ $=\cos\left(-\frac{\pi}{6}\right)$

   $\sin\frac{\pi}{6}=$ _________ $=\sin(2\pi+$ ______$)$.
3. sine function is non-negative in _______ and _______ quadrants.
4. cosine function is non-negative in ______ and _____ quadrants.

Application

1. The activity can be used to get the values for tan, cot, sec, and cosec functions also.
2. From this activity students may learn that
   sin (– θ) = – sin θ and cos (– θ) = – cos θ

   This activity can be applied to other trigonometric functions also.

Activity 9

Objective

To prepare a model to illustrate the values of sine function and cosine function for different angles which are multiples of $\frac{\pi}{2}$ and π.

Material Required

A stand fitted with 0º-360º protractor and a circular plastic sheet fixed with handle which can be rotated at the centre of the protractor.

Method of Construction

1. Take a stand fitted with 0º-360º protractor.
2. Consider the radius of protractor as 1 unit.

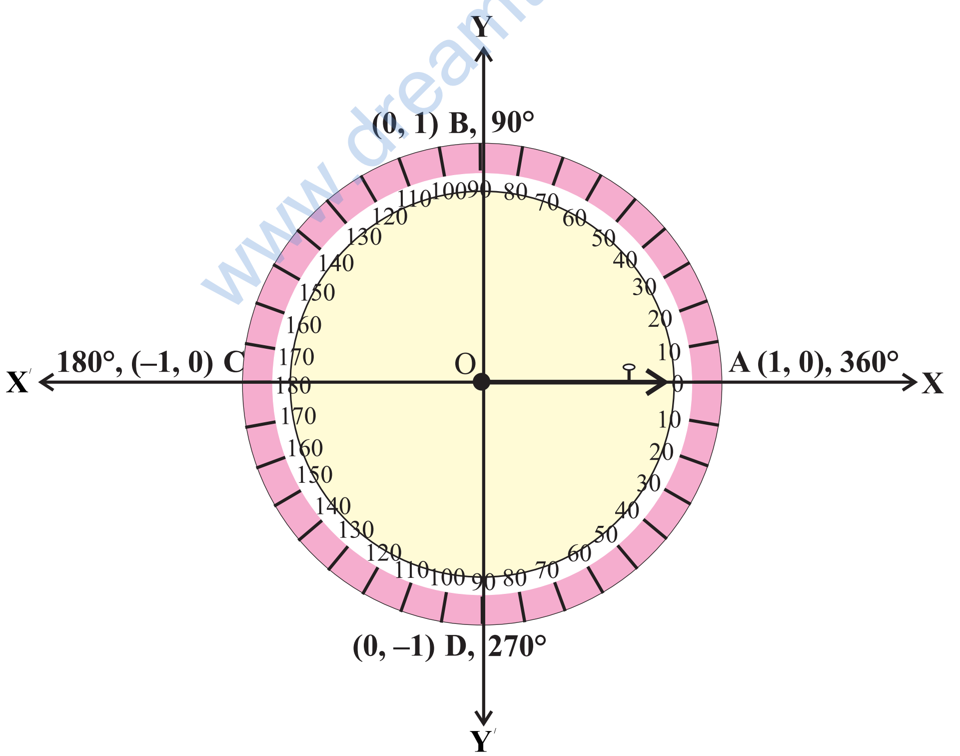

Fig. 9

3. Draw two lines, one joining 0º-180º line and another 90º-270º line, obviously perpendicular to each other.
4. Mark the ends of 0°-180° line as (1,0) at 0°, (–1, 0) at 180° and that of 90° - 270° line as (0,1) at 90° and (0, –1) at 270°
5. Take a plastic circular plate and mark a line to indicate its radius and fix a handle at the outer end of the radius.
6. Fix the plastic circular plate at the centre of the protractor.

Demonstration

1. Move the circular plate in anticlock wise direction to make different angles like 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π etc.
2. Read the values of sine and cosine function for these angles and their multiples from the perpendicular lines.

Observation

1. When radius line of circular plate is at 0º indicating the point A (1,0), cos 0 = ______ and sin 0 = _______.
2. When radius line of circular plate is at 90º indicating the point B (0, 1), $\cos\frac{\pi}{2}$ = _______ and $\sin\frac{\pi}{2}$ = _______ .
3. When radius line of circular plate is at 180º indicating the point C (–1,0), cos π = ______ and sin π = _________.
4. When radius line of circular plate is at 270º indicating the point D (0, – 1) which means $\cos\frac{3\pi}{2}$ = _______ and $\sin\frac{3\pi}{2}$ = _______
5. When radius line of circular plate is at 360º indicating the point again at A (1,0), cos 2 π = _______ and sin 2 π = ________.

Now fill in the table :

Trigonometric function	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$\sin\theta$	–	–	–	–	–	–	–	–	–
$\cos\theta$	–	–	–	–	–	–	–	–	–

APPLICATION

This activity can be used to determine the values of other trigonometric functions for angles being multiple of $\frac{\pi}{2}$ and π.

Activity 10

OBJECTIVE

To plot the graphs of sin x, sin $2x$, 2sin$x$ and sin$\frac{x}{2}$, using same coordinate axes.

MATERIAL REQUIRED

Plyboard, squared paper, adhesive, ruler, coloured pens, eraser.

METHOD OF CONSTRUCTION

1. Take a plywood of size 30 cm × 30 cm.
2. On the plywood, paste a thick graph paper of size 25 cm × 25 cm.
3. Draw two mutually perpendicular lines on the squared paper, and take them as coordinate axes.
4. Graduate the two axes as shown in the Fig. 10.
5. Prepare the table of ordered pairs for sin x, sin $2x$, 2sin x and sin $\frac{x}{2}$ for different values of x shown in the table below:

T. ratios	**0º**	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{9\pi}{12}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
sin x	0	0.26	0.50	0.71	0.86	0.97	1.00	0.97	0.86	0.71	0.50	0.26	0
sin $2x$	0	0.50	0.86	1.00	0.86	0.50	0	–0.5	–0.86	–1.0	–0.86	–0.50	0
2 sin x	0	0.52	1.00	1.42	1.72	1.94	2.00	1.94	1.72	1.42	1.00	0.52	0
sin $\frac{x}{2}$	0	0.13	0.26	0.38	0.50	0.61	0.71	0.79	0.86	0.92	0.97	0.99	1.00

DEMONSTRATION

1. Plot the ordered pair (x, sin x), (x, sin $2x$), (x, sin $\frac{x}{2}$) and (x, 2sin x) on the same axes of coordinates, and join the plotted ordered pairs by free hand curves in different colours as shown in the Fig.10.

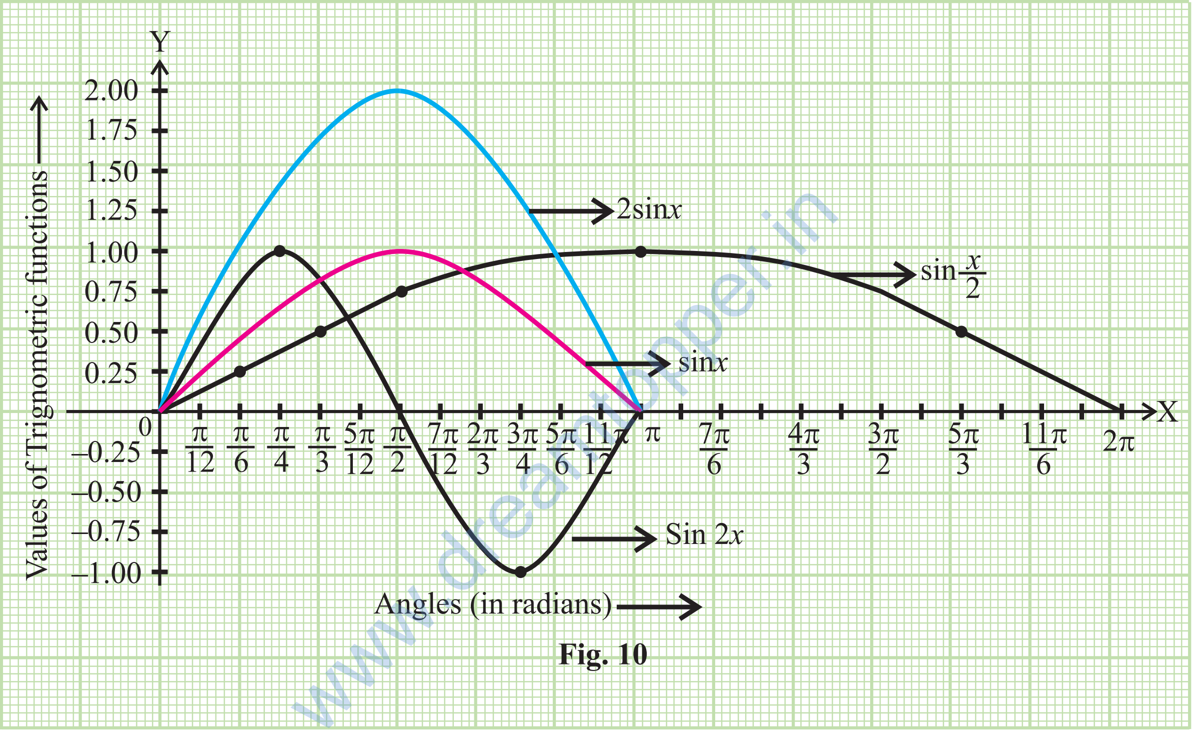

Fig. 10

OBSERVATION

1. Graphs of sin x and 2 sin x are of same shape but the maximum height of the graph of sin x is ________ the maximum height of the graph of ___________.
2. The maximum height of the graph of sin $2x$ is ___________. It is at x = __________.
3. The maximum height of the graph of 2 sin x is ___________ . It is at x = __________ .

4. The maximum height of the graph of sin $\frac{x}{2}$ is ___________ . It is at $\frac{x}{2}$ = _________ .

5. At x = ________, sin x = 0, at x = _______, sin $2x$ = 0 and at x = ________, sin $\frac{x}{2}$ = 0.

6. In the interval [0, π], graphs of sin x, 2 sin x and sin $\frac{x}{2}$ are _______ x - axes and some portion of the graph of sin $2x$ lies _______ x-axes.

7. Graphs of sin x and sin $2x$ intersect at x = _______ in the interval (0, π)

8. Graphs of sin x and sin $\frac{x}{2}$ intersect at x = ________ in the interval (0, π).

APPLICATION

This activity may be used in comparing graphs of a trigonometric function of multiples and submultiples of angles.